Day 138 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Position vectors and dividing a line in a ratio (All boards)

A has position vector $\begin{pmatrix}2\\1\end{pmatrix}$ and B has position vector $\begin{pmatrix}6\\1\end{pmatrix}$.
The point P lies on AB with AP : PB = 1 : 3.
Find the position vector of P.

$\text{AB} = \begin{pmatrix}4\\0\end{pmatrix}$

$\text{AP} = \frac{1}{4}\text{AB} = \begin{pmatrix}1\\0\end{pmatrix}$

$\text{OP} = \text{OA} + \text{AP} = \begin{pmatrix}3\\1\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{3}\\\mathbf{1}\end{pmatrix}$

2 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 5x^2 + kx$.
Given that $(x - 2)$ is a factor of f(x),
(a) find the value of k,
(b) factorise f(x) completely.

$f(2) = 0$: $8 - 20 + 2k = 0$
$k = 6$
$f(x) = (x - 2)(x^2 - 3x)$
$= (x - 3)(x - 2)x$
Answer: (a) k = 6 (b) f(x) = (x − 3)(x − 2)x

3 Trig equations in radians (All boards)

Solve $\tan x = -2$ for $0 \le x \le 2\pi$,
giving answers to 3 significant figures.

$\tan\theta = -2$ for $0 \le x \le 2\pi$
Calculator: $x = -1.107$, then use the symmetry of the graph
$x = 2.034, 5.176$
Answer: x = 2.03, 5.18

4 Integrating velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t seconds its velocity is $v = 4t^2 - 8t + 4$ m s^{-1}.
Find the distance moved between t = 1 and t = 4. (The particle does not change direction.)

$s = \int_1^4 (4t^2 - 8t + 4)\,dt$

$= [\,\frac{4}{3}t^3 - 4t^2 + 4t\,]$ from t = 1 to t = 4

$= 112/3 - 4/3$

Answer: 36 m

5 Conditional probability from a count (All boards)

50 people: 18 own a cat, 30 own a dog, 6 do neither.
(a) How many do both?
(b) Given a person who owns a cat, find P(this person owns a dog).

(a) At least one: $50 - 6 = 44$. Both $= 18 + 30 - 44 = 4$

(b) Of the 18 who own a cat, 4 also do the other: $\text{P} = \frac{4}{18} = \frac{2}{9}$

Answer: (a) 4 (b) $\mathbf{\frac{2}{9}}$

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6 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 25t - 2.5t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 25 - 5t$

Maximum speed when $a = 0$: $t = 25 \div 5 = 5$

$v = 25 \times 5 - 2.5 \times 5^2$

Answer: 62.5 m s^{-1} at t = 5 s

7 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = \frac{e^x}{x}$,

and determine its nature.

$\frac{dy}{dx} = \frac{(x-1)e^x}{x^2} = 0$

$x = 1$

$\frac{d^2y}{dx^2} = \frac{(x^2+2-2x)e^x}{x^3}$

At $x = 1$: $\frac{d^2y}{dx^2} = e > 0$, so a minimum

Answer: (1, e) minimum

8 Total probability and Bayes (All boards)

R: it rains; L: Ali is late.
P(R) = 0.4, P(L | R) = 0.2, P(L | R') = 0.2.
(a) Find P(L).
(b) Find P(R | L).

(a) $P(L) = 0.4 \times 0.2 + 0.6 \times 0.2 = 0.08 + 0.12 = 0.2$

(b) $P(R \mid L) = \frac{0.4 \times 0.2}{0.2} = \frac{0.08}{0.2} = 0.400$

Answer: (a) 0.2 (b) $\frac{2}{5}$

9 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{9x-21}{(x-4)(x-1)}$ in partial fractions.

$\frac{9x-21}{(x-4)(x-1)} \equiv \frac{A}{(x-4)} + \frac{B}{(x-1)}$, so $9x - 21 \equiv A(x-1) + B(x-4)$

$x = 4$: $15 = A \times 3$, so $A = 5$

$x = 1$: $-12 = B \times (-3)$, so $B = 4$

Answer: $\frac{5}{(x-4)} + \frac{4}{(x-1)}$

10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -x + 2$. The point P is $(-2, 2)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = -1, so the perpendicular has gradient 1

$y - 2 = (x + 2)$, giving $x - y + 4 = 0$

(b) Solve $y = -x + 2$ with $x - y + 4 = 0$: F = $(-1, 3)$

$PF^2 = (-1)^2 + (-1)^2 = 2$

Answer: (a) $x - y + 4 = 0$ (b) F$(-1, 3)$, PF = $\sqrt{2}$