Day 137 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = \frac{1}{(x^2 + 6)^2}$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = -1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = -\frac{4x}{(x^2 + 6)^3}$

When $x = -1$: gradient $= \frac{4}{343}$

Answer: $\frac{dy}{dx} = -\frac{4x}{(x^2 + 6)^3}$**; gradient** $\frac{4}{343}$

2 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 8 kg is pulled along rough horizontal ground by a horizontal force of 31.52 N. It accelerates at 1 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the crate and the ground.

$R = mg = 8 \times 9.8 = 78.4$
Resultant: $31.52 - F = 8 \times 1$, so $F = 23.52$
$F = \mu R$: $\mu = 23.52 \div 78.4$
Answer: 0.3

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 2\cos 5x\,dx$,

(b) the exact value of $\int_0^{\frac{\pi}{10}} 2\cos 5x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{2\sin 5x}{5} + c$

(b) $\left[\frac{2\sin 5x}{5}\right]_0^{\frac{\pi}{10}}$

$= \frac{2}{5}$

Answer: (a) $\frac{2\sin 5x}{5} + c$ **(b)** $\frac{2}{5}$

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(2x^3)^2 \times 2x}{8x}$.

$(2x^3)^2 = 4x^6$
Numerator $= 4 \times 2x^7 = 8x^7$
Divide: $\frac{8}{8} = 1$ and $x^7 \div x = x^6$

Answer: x^6

5 Product rule (All boards)

Given that $y = (x + 2)e^{-x}$,

find $\frac{dy}{dx}$, and its value when $x = 0$.

Product rule with $u = x + 2$, $v = e^{-x}$
$u' = 1$, $v' = -e^{-x}$
$\frac{dy}{dx} = u'v + uv' = (-x - 1)e^{-x}$

When $x = 0$: $\frac{dy}{dx} = -1$

Answer: $\frac{dy}{dx} = (-x - 1)e^{-x}$**; value** -1

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6 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 + px^2 + qx + 7$.
When $f(x)$ is divided by $(x - 3)$ the remainder is 58, and when divided by $(x + 1)$ the remainder is -2. Find p and q.

$f(3) = 58$: $54 + 9p + 3q + 7 = 58$, so $9p + 3q = -3$
$f(-1) = -2$: $-2 + p - q + 7 = -2$, so $p - q = -7$
Solving simultaneously: $p = -2$, $q = 5$
Answer: $p = -2$, $q = 5$

7 Binomial mean and variance (All boards)

X has a binomial distribution with mean 2 and variance 1.5.
(a) Find n and p.
(b) Find P(X = 2) to 3 significant figures.

(a) $np = 2$ and $np(1 - p) = 1.5$
$1 - p = 1.5 \div 2 = 0.75$, so $p = 0.25$ and $n = 2 \div 0.25 = 8$
(b) $P(X = 2) = {}^8C_2 \times 0.25^2 \times 0.75^6 = 0.311$
Answer: (a) $n = 8$, $p = 0.25$ (b) 0.311

8 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line with constant acceleration increases its speed from 3 m s^{-1} to 43 m s^{-1} while covering 230 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $1849 = 9 + 460\,a$, so $a = 4$
(b) $v = u + at$: $43 = 3 + 4t$
$t = 40 \div 4 = 10$
Answer: (a) 4 m s^{-2} (b) 10 s

9 Finding mean and standard deviation from two probabilities (All boards)

The scores on a test are normally distributed.
$P(X < 76) = 0.0228$ and $P(X > 112) = 0.1587$.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$76 = \mu + (-1.999)\sigma$ and $112 = \mu + 0.9998\sigma$
Subtract: $36 = 2.999\,\sigma$, so $\sigma = 12.0$
$\mu = 76 - (-1.999)(12.00) = 100$
Answer: $\mu = 100$, $\sigma = 12.0$

10 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 - 6x + 12y + 27 = 0$.
Find the coordinates of its centre and its radius.

Complete the square in x and y:
$(x - 3)^2 - 9 + (y + 6)^2 - 36 + 27 = 0$
$(x - 3)^2 + (y + 6)^2 = 18$
Centre $(3, -6)$, radius $3\sqrt{2}$
Answer: centre $(3, -6)$, radius $3\sqrt{2}$