Day 136 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-3 < 3x + 2 < 2$.

Subtract 2 from each part: $-5 < 3x < 0$

Divide each part by 3: $-\frac{5}{3} < x < 0$

Answer: $-\frac{5}{3} < x < 0$

2 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of values, $\bar{x} = 20$, $\bar{y} = 18$, $S_{xx} = 50$ and $S_{xy} = -25$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when $x = 24$.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-25}{50} = -0.5$

$a = \bar{y} - b\bar{x} = 18 - -0.5 \times 20 = 28$
$y = 28 - 0.5x$
(b) $y = 28 + (-0.5) \times 24 = 16$
Answer: $y = 28 - 0.5x$; $y = 16$

3 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(-\mathbf{i})$ m s^{-1} at time $t = 0$ and moves with constant acceleration $(-\mathbf{i} - 2\mathbf{j})$ m s^{-2}.
Find its velocity and speed when $t = 2$.

$v = u + at = (-\mathbf{i}) + 2(-\mathbf{i} - 2\mathbf{j})$
$= -3\mathbf{i} - 4\mathbf{j}$
Speed $= \sqrt{(-3^2 + -4^2)}$
Answer: $\mathbf{v} = (-3\mathbf{i} - 4\mathbf{j})$ m s^{-1}; speed 5 m s^{-1}

4 Reading a histogram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The histogram shows the times taken by a group of people to complete a task.
Estimate the number of people who took between 15 and 29 minutes.

Frequency = frequency density × class width.
15 to 25: $4 \times 10 = 40$
25 to 29: $4 \times 4 = 16$
Total = 56
Answer: 56

5 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $8\cos^2 x - 10\sin x - 11 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2 x = 1 - \sin^2 x$: $-8\sin^2 x - 10\sin x - 3 = 0$, i.e. $8\sin^2 x + 10\sin x + 3 = 0$
$(4\sin x + 3)(2\sin x + 1) = 0$, so $\sin x = -\frac{3}{4}$ or $-\frac{1}{2}$

$\sin x = -\frac{3}{4}$: $x = 228.6°, 311.4°$

$\sin x = -\frac{1}{2}$: $x = 210.0°, 330.0°$

Answer: $x = 210.0°, 228.6°, 311.4°, 330.0°$

6 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(9 + 3x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 9: $(9 + 3x)^{\frac{3}{2}} = 9^{\frac{3}{2}}(1 + \frac{1}{3}x)^{\frac{3}{2}} = 27(1 + \frac{1}{3}x)^{\frac{3}{2}}$

Expand $(1 + \frac{1}{3}x)^{\frac{3}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 27: coefficients of 1, x, x^2, x^3 are 27, $\frac{27}{2}$, $\frac{9}{8}$, $-\frac{1}{16}$

Valid when $|\frac{1}{3}x| < 1$, that is $|x| < 3$

Answer: (a) $27 + \frac{27}{2}x + \frac{9}{8}x^2 - \frac{1}{16}x^3 + ...$ (b) $|x| < 3$

7 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 3 : 1, and N lies on OB with ON : NB = 1 : 3.
Find OM and NM in terms of a and b.

AB = b − a, so AM = $\frac{3}{4}$(b − a)

OM = a + AM = $\frac{a}{4} + \frac{3b}{4}$

ON = $\frac{b}{4}$, so NM = OM − ON = $\frac{a}{4} + \frac{b}{2}$

Answer: OM = $\frac{a}{4} + \frac{3b}{4}$, NM = $\frac{a}{4} + \frac{b}{2}$

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line 12x + 5y = k is a tangent to the circle
$(x + 2)^2 + (y - 3)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (12, 5).
A unit step in this direction is (12, 5)/13, so the points of contact are
(−2, 3) ± 13 × (12, 5)/13 = (−2, 3) ± (12, 5)
T = (10, 8) or (−14, −2), and k = 12x + 5y gives:
k = 160 or k = −178
Answer: k = 160 or k = −178

9 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write 15 cos θ + 8 sin θ in the form R cos(θ − α), with R > 0 and 0 < α < 90°.
(b) Hence find its maximum value and the smallest θ ≥ 0 at which it occurs. Give α to 1 d.p.

R = $\sqrt{15^2 + 8^2}$ = 17, tan α = $\frac{8}{15}$ so α = 28.1°

Maximum is R when cos(θ − α) = 1, i.e. θ = α
Answer: (a) 17 cos(θ − 28.1°) (b) max 17 at θ = 28.1°

10 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 10 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where tan α = $\frac{4}{3}$ and tan β = $\frac{3}{4}$.

Take g = 9.8 m s^{-2}.
Find the tension in each string.

Horizontal: $T_1 \cos \alpha = T_2 \cos \beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin \alpha + T_2 \sin \beta = 98$

From the first: $T_1 = \frac{4}{3} T_2$; into the second: $T_2 \times \frac{5}{3} = 98$

Answer: T_1 = 78.4 N (at α); T_2 = 58.8 N (at β)