Day 135 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Integration by parts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use integration by parts to find the exact value of

$\int_1^2 x^2 \ln x \, dx$.

Let $u = \ln x$ and $\frac{dv}{dx} = x^2$, so $\frac{du}{dx} = \frac{1}{x}$ and $v = \frac{x^3}{3}$

Integral $= [uv] - \int v\frac{du}{dx}\,dx$

$= [\frac{x^3 \ln x}{3}]_1^2 - \int_1^2 \frac{x^2}{3}\,dx$

$= \frac{8 \ln 2}{3} - \frac{7}{9}$

Answer: $\frac{8 \ln 2}{3} - \frac{7}{9}$

2 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 3 kg is moving on a smooth horizontal surface at 0 m s^{-1} when a constant horizontal force of 6 N, in the direction of motion, acts for 2 s.

Find the speed of the puck at the end of this time and the distance it moves.

$F = ma$: $a = 6 \div 3 = 2$

$v = u + at = 0 + 2 \times 2$

$s = ut + \frac{1}{2}at^2$

Answer: 4 m s^{-1}; 4 m

3 Conditional probability from a count (All boards)

80 students: 23 play football, 11 play tennis, 52 do neither.

(a) How many do both?

(b) Given a person who plays football, find P(this person plays tennis).

(a) At least one: $80 - 52 = 28$. Both $= 23 + 11 - 28 = 6$

(b) Of the 23 who play football, 6 also do the other: $P = \frac{6}{23} = \frac{6}{23}$

Answer: (a) 6 (b) $\frac{6}{23}$

4 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 80 kg stands on the floor of a lift. The lift accelerates downwards at 0.6 m s^{-2}.

Take g = 9.8 m s^{-2}.

Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 80 \times 9.8 = -80 \times 0.6$

$R = 80(9.8 - 0.6)$

Answer: 736 N

5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 4x - 6$. Use the Newton-Raphson method with

$x_0 = 1$ to find x_1 and x_2, to 4 d.p.

$f'(x) = 3x^2 + 4$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 1 - \frac{-1.000}{7.000} = 1.1429$

$x_2 = 1.1429 - \frac{0.06414}{7.918} = 1.1348$

Answer: x_1 = 1.1429, x_2 = 1.1348

6 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 4| > 6$.

$|2x - 4| > 6$ means $2x - 4 < -6$ or $2x - 4 > 6$
Solve each part (dividing by 2): $x < -1$ or $x > 5$
Answer: $x < -1$ or $x > 5$

7 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^2$ for $0 \le x \le 5$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 2). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^3/3]$ from 0 to 5 $= k \times \frac{5^3}{3} = 1$, so $k = \frac{3}{125}$

(b) $P(X > 2) = \int_2^5 \frac{3}{125} x^2\,dx = 1 - \frac{2^3}{5^3} = \frac{117}{125}$

(c) $E(X) = \int_0^5 x\,f(x)\,dx = \frac{3}{125} \times \frac{5^4}{4} = \frac{15}{4}$

Answer: (a) $k = \frac{3}{125}$ (b) $\frac{117}{125}$ (c) $\frac{15}{4}$

8 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 26 \times 5^x + 25 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 26y + 25 = 0$
$(y - 1)(y - 25) = 0$
$y = 1$ or $y = 25$
$5^x = 1$ gives $x = 0$; $5^x = 25$ gives $x = 2$.
Answer: $x = 0$ or $x = 2$

9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{3x + 18}{(x + 2)(x + 5)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{3x + 18}{(x + 2)(x + 5)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{3x + 18}{(x + 2)(x + 5)} \equiv \frac{A}{x + 2} + \frac{B}{x + 5}$, so $3x + 18 \equiv A(x + 5) + B(x + 2)$
$x = -2$: $A = 4$; $x = -5$: $B = -1$

(b) $\int_0^4 \left(\frac{4}{x + 2} + \frac{-1}{x + 5}\right) dx = [4 \ln(x + 2) - \ln(x + 5)]_0^4$

$= 4 \ln \frac{6}{2} - \ln \frac{9}{5} = \ln(45)$

Answer: (a) $A = 4$, $B = -1$ (b) $\ln(45)$

10 Circle through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle passes through A(9, −6) and B(−7, −10), and its centre lies on the y-axis. Find the centre and the equation of the circle.

Let the centre be (0, h). Then $CA^2 = CB^2$:
$81 + (h + 6)^2 = 49 + (h + 10)^2$
Expanding and cancelling h^2: $12h + 117 = 20h + 149$
$h = -4$
$r^2 = CA^2 = 85$
Answer: centre $(0, -4)$, $x^2 + (y + 4)^2 = 85$