Day 134 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 41 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 41$, $\Sigma fx = 179$, $\Sigma fx^2 = 889$

Mean $= \frac{179}{41} = 4.37$

Variance $= \frac{889}{41} - (4.3659)^2 = 2.622$

Standard deviation = 1.62

Answer: mean 4.37, standard deviation 1.62

2 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\cot\theta = 3$ and θ is acute,
find the exact value of $\operatorname{cosec}\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\tan\theta = \frac{1}{3}$, so $\sec^2\theta = 1 + \frac{1}{9} = \frac{10}{9}$

$\cot\theta = 3$, so $\operatorname{cosec}^2\theta = 10$ and $\operatorname{cosec}\theta = \sqrt{10}$

Answer: $\sqrt{10}$

3 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-2, -1)$ and passes through the point $(-5, 2)$.
Find an equation of the circle.

$r^2 = (-5 - (-2))^2 + (2 - (-1))^2$
$= 9 + 9 = 18$

Answer: $(x + 2)^2 + (y + 1)^2 = 18$

4 Probability without replacement (All boards)

A bag has 4 white and 5 black sweets; two are taken without replacement.
(a) Find P(exactly one white).
(b) Find P(at least one white).

(a) P(white then black) + P(black then white) $= 2 \times \frac{4}{9} \times \frac{5}{8} = \frac{5}{9}$

(b) 1 − P(both black) $= 1 - \frac{5}{9} \times \frac{4}{8} = 1 - \frac{5}{18} = \frac{13}{18}$

Answer: (a) $\frac{5}{9}$ (b) $\frac{13}{18}$

5 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $21\cos\theta + 20\sin\theta \equiv 29\cos(\theta - 43.6°)$,
solve $21\cos\theta + 20\sin\theta = -5$ for $0 \le \theta \le 360°$. (1 d.p.)

$29\cos(\theta - 43.6°) = -5$, so $\cos(\theta - 43.6°) = \frac{-5}{29}$

$\theta - 43.6° = \pm 99.9°$
$\theta = 43.6° + 99.9° = 143.5°$
$\theta = 43.6° - 99.9° = -56.3°$, so $\theta = 303.7°$

Answer: θ = 143.5°, 303.7°

6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 - 4x + 2$.
C is stretched by scale factor 3 parallel to the y-axis.
Find its new equation.

Stretch parallel to the y-axis: multiply y by the scale factor.

When stretched by scale factor 3 parallel to the y-axis: $y = 3x^2 - 12x + 6$

Answer: $y = 3x^2 - 12x + 6$

7 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from a bridge, 4.9 m above the ground.
Take $g = 9.8$ m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $4.9 = \frac{1}{2} \times 9.8 \times t^2$
$t^2 = 4.9 \div 4.9 = 1$, so $t = 1$
(b) $v = u + at = 9.8 \times 1$
Answer: (a) 1 s (b) 9.8 m s^{-1}

8 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of
$(2 + 4x)(1 + 2x)^4$.

$(1 + 2x)^4$ has x^2 coefficient 24 and x^3 coefficient 32
x^3 term of the product $= 2 \times 32 + 4 \times 24$
$= 160$
Answer: 160

9 Differentiating displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on a straight line. Its displacement from O at time t seconds is $s = 4t^3 - t^2 + t$ metres.
Find its velocity and acceleration when $t = 3$.

$v = ds/dt = 12t^2 - 2t + 1$
$a = dv/dt = 24t - 2$
$t = 3$: $v = 103$, $a = 70$
Answer: v = 103 m s^{-1}, a = 70 m s^{-2}

10 Quotient rule (All boards)

Given that $y = \frac{x^2 + 4}{4x + 3}$,

find $\frac{dy}{dx}$, and its value when $x = 3$.

Quotient rule with $u = x^2 + 4$, $v = 4x + 3$
$u' = 2x$, $v' = 4$
$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{2(x(4x + 3) - 2x^2 - 8)}{(4x + 3)^2}$$
When $x = 3$: $\frac{dy}{dx} = \frac{38}{225}$
Answer: $\frac{dy}{dx} = \frac{2(x(4x + 3) - 2x^2 - 8)}{(4x + 3)^2}$; value $\frac{38}{225}$