**Day 133** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle $\theta$ above the positive x-axis, a force of 16 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{4}{5}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 16$, so $P \times \frac{4}{5} = 16$

$P = 20$

Resolve vertically: $Q = P\sin\theta = 20 \times \frac{3}{5}$

**Answer: P = 20 N; Q = 12 N**

## 2 Transformations of a turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

y = f(x) has a minimum point at (6, 1).
Find the turning point on the curve:
(a) y = f(x) − 2 (b) y = f(x + 1)

(c) $y = f(\frac{x}{3})$

(a) y = f(x) − 2: the point (6, 1) maps to (6, −1)
(b) y = f(x + 1): the point (6, 1) maps to (5, 1)

(c) $y = f(\frac{x}{3})$: the point (6, 1) maps to (18, 1)

**Answer: (a) (6, −1) (b) (5, 1) (c) (18, 1)**

## 3 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A ship moves with constant velocity $(-2\mathbf{i} + \mathbf{j})$ m s$^{-1}$. At time t = 0 its position vector relative to a fixed origin O is $(-4\mathbf{i} - 2\mathbf{j})$ m.
Find (a) its position vector when t = 3, (b) the value of t when it crosses the line through O in the direction of **i**.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-4\mathbf{i} - 2\mathbf{j}) + 3(-2\mathbf{i} + \mathbf{j}) = (-10\mathbf{i} + \mathbf{j})$
(b) **j** component zero: $-2 + t = 0$, so $t = 2$
**Answer: (a) (−10i + j) m (b) t = 2 s**

## 4 Quotient rule (All boards)

Given that $y = \frac{\cos x}{x^2}$,

find $\frac{dy}{dx}$, and its value when $x = \pi$.

Quotient rule with $u = \cos x$, $v = x^2$
$u' = -\sin x$, $v' = 2x$
$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = -\frac{x\sin x + 2\cos x}{x^3}$$

When $x = \pi$: $\frac{dy}{dx} = \frac{2}{\pi^3}$

**Answer:** $\frac{dy}{dx} = -\frac{x\sin x + 2\cos x}{x^3}$**; value** $\frac{2}{\pi^3}$

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## 5 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf f(x) = kx for $0 \leq x \leq 5$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 4). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^2/2]$ from 0 to 5 = $k \times \frac{5^2}{2} = 1$, so $k = \frac{2}{25}$

(b) $P(X > 4) = \int_4^5 \frac{2}{25} x\,dx = 1 - \frac{4^2}{5^2} = \frac{9}{25}$

(c) $E(X) = \int_0^5 x\,f(x)\,dx = \frac{2}{25} \times \frac{5^3}{3} = \frac{10}{3}$

**Answer: (a) k = $\frac{2}{25}$ (b) $\frac{9}{25}$ (c) $\frac{10}{3}$**

## 6 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $10\sin^2 x + 9\sin x - 9 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

$(2\sin x + 3)(5\sin x - 3) = 0$, so $\sin x = -\frac{3}{2}$ or $\sin x = \frac{3}{5}$

$\sin x = -\frac{3}{2}$ has no solutions

$\sin x = \frac{3}{5}$: x = 36.9°, 143.1°

**Answer: x = 36.9°, 143.1°**

## 7 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 + x - 2$.
C is translated by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and then stretched by scale factor 2 parallel to the y-axis.
Find its new equation.

Translation: replace x by $(x - h)$ and add k to y.
When translated by $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$: $y = x^2 - 3x - 1$
Stretch parallel to the y-axis: multiply y by the scale factor.
When stretched by scale factor 2 parallel to the y-axis: $y = 2x^2 - 6x - 2$

**Answer: $y = 2x^2 - 6x - 2$**

## 8 Binomial mean and variance (All boards)

X has a binomial distribution with mean 6 and variance 4.2.
(a) Find n and p.
(b) Find P(X = 6) to 3 significant figures.

(a) $np = 6$ and $np(1 - p) = 4.2$
$1 - p = 4.2 \div 6 = 0.7$, so $p = 0.3$ and $n = 6 \div 0.3 = 20$
(b) $P(X = 6) = {}^{20}C_6 \times 0.3^6 \times 0.7^{14} = 0.192$

**Answer: (a) n = 20, p = 0.3 (b) 0.192**

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## 9 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{-2x-6}{(x+2)(x+4)}$ in partial fractions.

(b) Hence evaluate $\int_0^4 \frac{-2x-6}{(x+2)(x+4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{-2x-6}{(x+2)(x+4)} \equiv \frac{A}{x+2} + \frac{B}{x+4}$, so $-2x-6 \equiv A(x+4) + B(x+2)$
$x = -2$: $A = -1$; $x = -4$: $B = -1$

(b) $\int_0^4 \left(\frac{-1}{x+2} + \frac{-1}{x+4}\right) dx = [-\ln(x+2) - \ln(x+4)]_0^4$
$= -\ln\frac{6}{2} - \ln\frac{8}{4} = \ln\left(\frac{1}{6}\right)$

**Answer: (a) $A = -1$, $B = -1$ (b) $\ln\left(\frac{1}{6}\right)$**

## 10 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$, a quadratic.
(a) Find the coordinates of the local maximum point of $y = |f(x)|$.
(b) How many solutions has $|f(x)| = 12$?

(a) $y = |f(x)|$ reflects the part of the curve below the x-axis in the x-axis:
$(3, -16)$ becomes $(3, 16)$
(b) The maximum of the middle part is 16. Compare $k = 12$ with 16:
$k < 16$ gives 4 intersections with $y = k$.
**Answer: (a) (3, 16) (b) 4**