Day 132 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Union, conditional probability and independence (All boards)

P(A) = 0.6, P(B) = 0.2, P(A ∩ B) = 0.1.
Find (a) P(A ∪ B), (b) P(A | B).
(c) Are A and B independent?

(a) $P(A \cup B) = 0.6 + 0.2 - 0.1 = 0.7$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.2} = 0.5$

(c) $P(A)P(B) = 0.6 \times 0.2 = 0.12$, which does not equal $P(A \cap B)$

Answer: (a) 0.7 (b) 0.5 (c) not independent

2 Free fall from rest (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coin is dropped from rest from a bridge, 44.1 m above the ground.
Take g = 9.8 m s^{-2}.
Find (a) the time taken to reach the ground, (b) its speed on landing.

(a) $s = ut + \frac{1}{2}at^2$: $44.1 = \frac{1}{2} \times 9.8 \times t^2$

$t^2 = 44.1 \div 4.9 = 9$, so $t = 3$

(b) $v = u + at = 9.8 \times 3$

Answer: (a) 3 s (b) 29.4 m s^{-1}

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = -3x - 6$ and $g(x) = x^2 - 5$ for $x \in \mathbb{R}$.
(a) Find fg(x).
(b) Solve gf(x) = 31.

$fg(x) = f(x^2 - 5) = -3(x^2 - 5) - 6 = -3x^2 + 9$

$gf(x) = (-3x - 6)^2 - 5 = 31$

$(-3x - 6)^2 = 36$, so $-3x - 6 = \pm 6$

$x = -4$ or $x = 0$

Answer: (a) $fg(x) = -3x^2 + 9$ (b) $x = -4$ or $x = 0$

4 Tree diagram (All boards)

The tree diagram shows probabilities for event E, followed by event F.
Find the probability that exactly one of E and F occurs.

P(E and not F) = 0.4 × 0.2 = 0.08
P(not E and F) = 0.6 × 0.9 = 0.54
Exactly one: 0.08 + 0.54 = 0.62
Answer: 0.62

5 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 1.5 cm per second.
Find the rate at which its area is increasing when r = 5 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 1.5$

When r = 5: $\frac{dA}{dt} = 15\pi$

Answer: 15π cm^2/s

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6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $3x - 7y = -3$ and line l_2 has equation $x - 4y = -6$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = (6, 3)
l_1 meets y = 0 at $x = -1$, and l_2 meets y = 0 at $x = -6$
Base = 5, height = 3
Area = $\frac{1}{2} \times$ base $\times$ height = $\frac{15}{2}$

Answer: $\frac{15}{2}$

7 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is held on a rough plane ($\tan\alpha = \frac{4}{3}$, $\mu = 0.25$) by a force P N up the plane, as shown.
Take g = 9.8 m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 49 \times 3/5 = 29.4$
Friction (limiting) $F = \mu R = 7.35$
Moving up, friction acts down: $P = mg\sin\alpha + F$
$mg\sin\alpha = 39.2$
Answer: 46.55 N

8 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t + 1$, $y = -t^2 - 4t - 5$, $t \in \mathbb{R}$.
Find a Cartesian equation of the curve in the form y = f(x).

From the first equation: $t = x - 1$
Substitute t into $y = -t^2 - 4t - 5$ and expand.
$y = -x^2 - 2x - 2$
Answer: $y = -x^2 - 2x - 2$

9 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass M kg of a substance satisfies $\frac{dM}{dt} = -0.2M$, where t is in years.
Initially M = 50. Find M when t = 10, and the half-life, each to 3 s.f.

$\frac{1}{M}\,dM = -0.2\,dt$, so $\ln M = -0.2t + c$, giving $M = 50e^{-0.2t}$
$t = 10$: $M = 50\,e^{-2} = 6.767$
Half-life: $e^{-0.2t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.2} = 3.466$
Answer: M = 6.77 kg, half-life 3.47 years

10 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x + 4}{(x + 1)(x + 3)}$ in partial fractions.
(b) Hence evaluate $\int_0^4 \frac{2x + 4}{(x + 1)(x + 3)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x + 4}{(x + 1)(x + 3)} \equiv \frac{A}{x + 1} + \frac{B}{x + 3}$, so $2x + 4 \equiv A(x + 3) + B(x + 1)$
$x = -1$: A = 1; $x = -3$: B = 1
(b) $\int_0^4 \left(\frac{1}{x + 1} + \frac{1}{x + 3}\right) dx = [\ln(x + 1) + \ln(x + 3)]_0^4$
$= \ln\frac{5}{1} + \ln\frac{7}{3} = \ln\left(\frac{35}{3}\right)$

Answer: (a) A = 1, B = 1 (b) $\ln\left(\frac{35}{3}\right)$