Day 131 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial probabilities (All boards)

X ~ B(15, 0.4). Find, to 3 significant figures,
(a) P(X = 7)
(b) P(X ≤ 7)

(a) $P(X = 7) = {}^{15}C_7 \times 0.4^7 \times 0.6^8 = 0.177$

(b) P(X ≤ 7) from the cumulative binomial function

(b) = 0.787

Answer: (a) 0.177 (b) 0.787

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates (1, 4) and (8, 5).
(a) Find the midpoint of AB.
(b) Find the line AB as ax + by + c = 0, with integer a, b, c.

(a) Midpoint $= \left(\frac{1+8}{2}, \frac{4+5}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

(b) Gradient $= \frac{1}{7}$

$y - 4 = \frac{1}{7}(x - 1)$

Multiply out and collect terms: $x - 7y + 27 = 0$

Answer: (a) $\left(\frac{9}{2}, \frac{9}{2}\right)$ (b) $x - 7y + 27 = 0$

3 Product rule (All boards)

Given that $y = e^x \cos x$,

find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{2}$.

Product rule with $u = e^x$, $v = \cos x$

$u' = e^x$, $v' = -\sin x$

$\frac{dy}{dx} = u'v + uv' = (\cos x - \sin x)e^x$

When $x = \frac{\pi}{2}$: $\frac{dy}{dx} = -e^{\frac{\pi}{2}}$

Answer: $\frac{dy}{dx} = (\cos x - \sin x)e^x$; value $-e^{\frac{\pi}{2}}$

4 Poisson: changing the interval (CIE)

Customers arrive at a shop at random at an average rate of 8 per hour.
Find the probability that in 30 minutes there are exactly 1 (3 s.f.).

Rate in 30 minutes: $\lambda = 8 \times \frac{30}{60} = 4$

X ~ Po(4)

$P(X = 1) = e^{-4} \times \frac{4^1}{1!} = 0.0733$

Answer: 0.0733

5 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^4 in the expansion of $(3 + x)^6$.

Term in $x^4 = 15 \times 3^2 \times (x)^4$

$= 15 \times 9 \times 1\ x^4$

Answer: 135

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6 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $(-4\mathbf{i} - 5\mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(2\mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

$\mathbf{F} = ma$: $(-4\mathbf{i} - 5\mathbf{j}) + \mathbf{P} = 4(2\mathbf{j}) = (8\mathbf{j})$
$\mathbf{P} = 4\mathbf{i} + 13\mathbf{j}$
$|\mathbf{P}| = \sqrt{(4^2 + 13^2)}$
Answer: $\mathbf{P} = (4\mathbf{i} + 13\mathbf{j})$ N; $|\mathbf{P}| = 13.6$ N

7 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{5y}{x}$ for $x > 0$,
given that $y = 4$ when $x = 1$.

$\frac{1}{y}\,dy = \frac{5}{x}\,dx$, so $\ln y = 5 \ln x + c$
$x = 1$, $y = 4$: $c = \ln 4$
$y = 4x^5$
Answer: $y = 4x^5$

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $12x - 5y = k$ is a tangent to the circle
$(x - 3)^2 + (y - 3)^2 = 169$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(12, -5)$.
A unit step in this direction is $(12, -5)/13$, so the points of contact are
$(3, 3) \pm 13 \times (12, -5)/13 = (3, 3) \pm (12, -5)$
T = $(15, -2)$ or $(-9, 8)$, and $k = 12x - 5y$ gives:
$k = 190$ or $k = -148$
Answer: $k = 190$ or $k = -148$

9 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 10 kg is pulled across smooth horizontal ice by a rope with tension 26 N, inclined at α above the horizontal, where $\tan\alpha = \frac{12}{5}$.

Take $g = 9.8$ m s^{-2}.
Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos\alpha = 5/13$, $\sin\alpha = 12/13$
(a) Horizontal: $26 \times 5/13 = 10\,a$
(b) Vertical: $R + 26 \times 12/13 = 10 \times 9.8$
Answer: (a) 1 m s^{-2} (b) 74 N

10 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 12 cm and area 54π cm^2.
Find the angle of the sector in degrees.

$\frac{1}{2}r^2\theta = 54\pi$, so $\theta = \frac{2 \times 54\pi}{12^2} = \frac{3\pi}{4}$ radians
$\theta = \frac{3\pi}{4} \times \frac{180}{\pi} = 135°$
Answer: 135°