**Day 130** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

20 pairs of rainfall and crop yield give $r = -0.264$.
One-tailed critical value (5%): 0.3783.
Test for negative correlation.

$H_0: \rho = 0$, $H_1: \rho < 0$
$r = -0.264$, critical value $-0.3783$; r is not beyond the critical value in the right direction
r is not in the critical region, so do not reject $H_0$
Conclusion: there is insufficient evidence of negative correlation between rainfall and crop yield
**Answer: do not reject $H_0$: there is insufficient evidence of negative correlation between rainfall and crop yield**

## 2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_1^2 (3x^2 + \frac{3}{x})\,dx$.

$\int (3x^2 + \frac{3}{x})\,dx = x^3 + 3\ln x$
$= [x^3 + 3\ln x]_1^2$
$= \ln 8 + 7$
**Answer: ln 8 + 7**

## 3 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 3rd term 37 and 12th term 118.
(a) Find the first term and the common difference.
(b) Find the sum of the first 15 terms.

$a + 2d = 37$ and $a + 11d = 118$
Subtract: $9d = 81$, so $d = 9$ and $a = 19$
$S_{15} = \frac{15}{2}(2 \times 19 + 14 \times 9) = 1230$
**Answer: (a) a = 19, d = 9 (b) 1230**

## 4 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sports car starts from rest and accelerates at 2 m $s^{-2}$. At that moment it is passed by a coach travelling at constant speed 9 m $s^{-1}$.
Find how long it takes to catch up, and the distance travelled.

Same distance: $\frac{1}{2} \times 2 \times t^2 = 9t$
$t(1t - 9) = 0$, $t \neq 0$ so $t = 9 \div 1$
Distance $= 9 \times 9$
**Answer: 9 s, 81 m**

## 5 Combined inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x for which
$4x + 4 \geq -5$ and $x^2 + x - 30 > 0$.

$4x + 4 \geq -5$ gives $x \geq -\frac{9}{4}$
$x^2 + x - 30 > 0$: $(x + 6)(x - 5) > 0$ gives $x < -6$ or $x > 5$
Both must hold: $x > 5$
**Answer: x > 5**

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## 6 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle $\theta$ above the positive x-axis, a force of 60 N in the negative x-direction and a force of Q N in the negative y-direction.

$\cos\theta = \frac{15}{17}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 60$, so $P \times \frac{15}{17} = 60$

$P = 68$

Resolve vertically: $Q = P\sin\theta = 68 \times \frac{8}{17}$

**Answer: P = 68 N; Q = 32 N**

## 7 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - e^{x} - 6 = 0$,
giving any answers as exact values.

Let $y = e^{x}$: $y^2 - y - 6 = 0$
$(y + 2)(y - 3) = 0$
$e^{x} > 0$, so $y = -2$ is rejected; $e^{x} = 3$
$x = \ln 3$
**Answer: x = ln 3**

## 8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(100, 0.6). Use a normal approximation to find $P(X \leq 58)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(60.00, 24.00)$, so $\sigma = \sqrt{24.00} = 4.899$
Use a continuity correction.
$P(X \leq 58) \approx P(Y < 58.5) = P(Z < -0.3062) = 0.380$
**Answer: 0.380**

## 9 Intersection of two lines in 3D (CIE)

Line $l_1$: $\mathbf{r} = \begin{pmatrix} 2 \\ 7 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

Line $l_2$: $\mathbf{r} = \begin{pmatrix} -5 \\ 2 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$.

Show that $l_1$ and $l_2$ intersect and find the point.

i: $2 + 1\lambda = -5 + 3\mu$
j: $7 + 3\lambda = 2 + 1\mu$
Solving i and j: $\lambda = -1$, $\mu = 2$
k check: $-4 + (-2)(-1) = -2$ and $-2 + 0(2) = -2$, equal
Point: $\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$
**Answer:** $\begin{pmatrix} \mathbf{1} \\ \mathbf{4} \\ \mathbf{-2} \end{pmatrix}$

## 10 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.

(a) Find the coordinates of the local maximum point of y = |f(x)|.

(b) How many solutions has |f(x)| = 10?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(3, −9) becomes (3, 9)

(b) The maximum of the middle part is 9. Compare k = 10 with 9:
k > 9 gives 2 intersections with y = k.

**Answer: (a) (3, 9) (b) 2**