Day 13 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chain rule (All boards)

Given that $y = e^{x^2 - x}$,

find $\frac{dy}{dx}$, and the gradient of the curve when $x = 0$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = (2x - 1)e^{x^2 - x}$

When $x = 0$: gradient $= -1$

Answer: $\frac{dy}{dx} = (2x - 1)e^{x^2 - x}$; gradient -1

2 Binomial probabilities (All boards)

$X \sim B(10, 0.7)$. Find, to 3 significant figures,

(a) $P(4 \leq X \leq 7)$

(b) $P(X > 6)$

(a) $P(4 \leq X \leq 7) = P(X \leq 7) - P(X \leq 3)$

(a) $= 0.607$

(b) $P(X > 6) = 1 - P(X \leq 6)$

(b) $= 0.650$

Answer: (a) 0.607 (b) 0.650

3 Product rule (All boards)

Given that $y = e^{-3x} \cos x$,

find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{2}$.

Product rule with $u = e^{-3x}$, $v = \cos x$

$u' = -3e^{-3x}$, $v' = -\sin x$

$\frac{dy}{dx} = u'v + uv' = (-\sin x - 3\cos x)e^{-3x}$

When $x = \frac{\pi}{2}$: $\frac{dy}{dx} = -\frac{1}{e^{\frac{3\pi}{2}}}$

Answer: $\frac{dy}{dx} = (-\sin x - 3\cos x)e^{-3x}$; value $-\frac{1}{e^{\frac{3\pi}{2}}}$

4 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 + 30x + 66$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 3x^2 + 30x + 66$.

Take out the factor 3: $3x^2 + 30x + 66 = 3(x^2 + 10x) + 66$

$= 3[(x + 5)^2 - 25] + 66$

$= 3(x + 5)^2 - 9$

$(x + 5)^2 \geq 0$ with equality at $x = -5$, so the minimum is -9 at $x = -5$.

Answer: (a) $3(x + 5)^2 - 9$ (b) $(-5, -9)$

5 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 2 m s^{-1} to 6.5 m s^{-1} while covering 38.25 m.

Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: $42.25 = 4 + 76.5\,a$, so $a = 0.5$

(b) $v = u + at$: $6.5 = 2 + 0.5\,t$

$t = 4.5 \div 0.5 = 9$

Answer: (a) 0.5 m s^{-2} (b) 9 s

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 7?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(0, −4) becomes (0, 4)
(b) The maximum of the middle part is 4. Compare k = 7 with 4:
k > 4 gives 2 intersections with y = k.
Answer: (a) (0, 4) (b) 2

7 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 6 m and mass 12 kg hangs horizontally from two vertical strings attached 1 m from A and 1 m from B.
A particle of mass 2 kg is attached 1.5 m from A. Take g = 9.8 m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(4) = 12\,g(2) + 2\,g(0.5)$
$T_2 = 61.25$
$T_1 + T_2 = 14\,g$ so $T_1 = 137.2 - 61.25$
Answer: Near A: 75.95 N; near B: 61.25 N

8 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = x^2 + 2$, the x-axis
and the lines x = 0 and x = 1.
R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_0^1 y^2\,dx = \pi \int_0^1 (x^4 + 4x^2 + 4)\,dx$
$= \pi\left[\frac{x^5}{5} + \frac{4x^3}{3} + 4x\right]_0^1$
$= \frac{83\pi}{15}$
Answer: $\frac{83\pi}{15}$

9 Binomial: smallest n (All boards)

Given that each trap catches a rabbit independently with probability 0.15, find the least number of traps needed for the probability of at least one success to exceed 0.999.

P(at least one) = $1 - 0.85^n > 0.999$
$0.85^n < 0.001$
$n > \frac{\log 0.001}{\log 0.85} = 42.50$
Answer: n = 43

10 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation y = −3x + 10. The point P is (0, 0).
(a) Find the line through P perpendicular to l, as ax + by + c = 0 with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l = −3, so the perpendicular has gradient $\frac{1}{3}$
$y = \frac{1}{3}x$, giving $x - 3y = 0$
(b) Solve $y = -3x + 10$ with $x - 3y = 0$: F = (3, 1)
$PF^2 = (-3)^2 + (-1)^2 = 10$
Answer: (a) x − 3y = 0 (b) F(3, 1), PF = $\sqrt{10}$