**Day 128** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Solving index equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{-2x+3} = 4^{-x+3}$.

Write both sides as powers of 2: $2^{3(-2x+3)} = 2^{2(-x+3)}$

Equate the indices: $-6x + 9 = -2x + 6$

$-4x = -3$

$x = \frac{3}{4}$

**Answer: $x = \frac{3}{4}$**

## 2 Standard integrals (definite) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (5 \sin 2x + 4 \cos 2x)\, dx$.

$\int (5 \sin 2x + 4 \cos 2x)\, dx = 2 \sin 2x - \frac{5 \cos 2x}{2}$

$= [2 \sin 2x - \frac{5 \cos 2x}{2}]_0^{\frac{\pi}{2}}$

$= 5$

**Answer: 5**

## 3 Conditional probability from a count (All boards)

60 members: 30 swim, 18 cycle, 15 do neither.

(a) How many do both?

(b) Given a person who cycles, find P(this person swims).

(a) At least one: $60 - 15 = 45$. Both $= 30 + 18 - 45 = 3$

(b) Of the 18 who cycle, 3 also do the other: $P = \frac{3}{18} = \frac{1}{6}$

**Answer: (a) 3 (b) $\frac{1}{6}$**

## 4 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations

$x = 4 + 3\cos t$, $y = -2 + 3\sin t$, for $0 \le t < 2\pi$. Find a Cartesian equation of the curve.

$\cos t = \frac{x - 4}{3}$ and $\sin t = \frac{y + 2}{3}$

Use $\cos^2 t + \sin^2 t = 1$:

$(x - 4)^2 + (y + 2)^2 = 9$

**Answer: $(x - 4)^2 + (y + 2)^2 = 9$**

## 5 Solving exponential equations using logarithms (All boards)

Solve $7^x = 37$, giving your answer to 3 s.f.

Take logs: $x \ln 7 = \ln 37$

$x = \frac{\ln 37}{\ln 7} = 1.856$

**Answer: $x = 1.86$**

## 6 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 2 m and mass 12 kg is hinged at A and held horizontal by a light string attached at B.

The string makes an angle $\theta$ with the rod, where $\sin\theta = \frac{3}{5}$. Take $g = 9.8$ m s$^{-2}$. Find the tension in the string.

Moments about A: $T \sin\theta \times 2 = mg \times 1$

$T \times \frac{3}{5} \times 2 = 117.6 \times 1$

$T = 117.6 \div 1.2$

**Answer: 98 N**

**Day 128** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $20\cos\theta + 21\sin\theta \equiv 29\cos(\theta - 46.4°)$,
solve $20\cos\theta + 21\sin\theta = -4.5$ for $0 \le \theta \le 360°$. (1 d.p.)

$29\cos(\theta - 46.4°) = -4.5$, so $\cos(\theta - 46.4°) = \frac{-4.5}{29}$

$\theta - 46.4° = \pm 98.9°$
$\theta = 46.4° + 98.9° = 145.3°$
$\theta = 46.4° - 98.9° = -52.5°$, so $\theta = 307.5°$
**Answer: $\theta$ = 145.3°, 307.5°**

## 8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers
m and n such that $15m + 25n = 114$.

Assume there are integers m and n with $15m + 25n = 114$.
$15m + 25n = 5(3m + 5n)$.
So the left-hand side is a multiple of 5.
But $114 = 5 \times 22 + 4$ is not a multiple of 5.
This is a contradiction, so no such integers exist.
**Answer: no integer solutions (LHS is a multiple of 5, 114 is not)**

## 9 Beam about to tilt (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform plank AB of length 4 m and mass 40 kg rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m.
A girl of mass 20 kg walks from D towards B. Find how far she is from D when the plank is about to tilt.

About to tilt about D: reaction at C is zero
Moments about D: $40\text{ g} \times (2.5 - 2) = 20\text{ g} \times x$
$40 \times 0.5 = 20x$
**Answer: 1 m**

## 10 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(120, 0.4)$. Use a normal approximation to find $P(X \ge 43)$, to 3 significant figures.

$Y \sim N(np, np(1-p)) = N(48.00, 28.80)$, so $\sigma = \sqrt{28.80} = 5.367$
Use a continuity correction.
$P(X \ge 43) \approx P(Y > 42.5) = P(Z > -1.025) = 0.847$
**Answer: 0.847**