Day 127 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - e^x - 12 = 0$,
giving any answers as exact values.

Let $y = e^x$: $y^2 - y - 12 = 0$
$(y + 3)(y - 4) = 0$
$e^x > 0$, so $y = -3$ is rejected; $e^x = 4$
$x = \ln 4$
Answer: $x = \ln 4$

2 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

8 values of x are coded using $y = \frac{(x - 100)}{5}$.

$\Sigma y = 9$ and $\Sigma y^2 = 239$.
Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 9 \div 8 = 1.125$
Mean of $x = 100 + 5 \times 1.125 = 106$
Variance of $y = \frac{239}{8} - (1.125)^2 = 28.61$
SD of $y = 5.349$, so SD of $x = 5 \times 5.349 = 26.7$
Answer: mean 106, standard deviation 26.7

3 Braking distance and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 10 m s^{-1} brakes and comes to rest with constant deceleration 4 m s^{-2}.
Find (a) the distance travelled while braking, (b) the time taken to stop.

(a) $0 = 10^2 - 2 \times 4 \times s$
$s = 100 \div 8 = 12.5$
(b) $0 = 10 - 4t$, so $t = 10 \div 4$
Answer: (a) 12.5 m (b) 2.5 s

4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when
$-x^3 + 2x^2 + 10x + 8$ is divided by $(x + 1)$.

$-x^2 \times (x + 1) = -x^3 - x^2$, leaving $3x^2 + 10x + 8$
$3x \times (x + 1) = 3x^2 + 3x$, leaving $7x + 8$
$7 \times (x + 1) = 7x + 7$, leaving 1
Quotient $-x^2 + 3x + 7$, remainder 1
Answer: quotient $-x^2 + 3x + 7$, remainder 1

5 Conditional probability from a count (All boards)

60 customers: 26 buy coffee, 23 buy tea, 18 do neither.
(a) How many do both?
(b) Given a person who buys coffee, find P(this person buys tea).

(a) At least one: $60 - 18 = 42$. Both $= 26 + 23 - 42 = 7$
(b) Of the 26 who buy coffee, 7 also do the other: $P = \frac{7}{26} = \frac{7}{26}$
Answer: (a) 7 (b) $\frac{7}{26}$

6 Rod hinged at one end and held by a string (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 2 m and mass 6 kg is hinged at A and held horizontal by a light string attached at B. The string makes an angle θ with the rod, where $\sin\theta = \frac{3}{5}$. Take $g = 9.8$ m s^{-2}. Find the tension in the string.

Moments about A: $T \sin\theta \times 2 = mg \times 1$

$T \times \frac{3}{5} \times 2 = 58.8 \times 1$

$T = 58.8 \div 1.2$

Answer: 49 N

7 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 10°C: $\frac{dT}{dt} = -k(T - 10)$. Initially $T = 70$ and after 5 minutes $T = 60$. Find k to 3 s.f. and the time to cool to 20°C, to 3 s.f.

$\frac{1}{T-10}\,dT = -k\,dt$, so $\ln(T - 10) = -kt + c$, i.e. $T = 10 + 60e^{-kt}$

$t = 5$: $60 = 10 + 60e^{-5k}$, so $e^{-5k} = \frac{5}{6}$

$k = \frac{1}{5}\ln\frac{60}{50} = 0.03646$

$20 = 10 + 60e^{-kt}$: $t = \frac{1}{k}\ln\frac{60}{10} = 49.14$

Answer: k = 0.0365, t = 49.1 minutes

8 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1-u_n}$, $u_1 = -1$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{25} and $\sum_{r=1}^{25} u_r$.

(a) $u_2 = \frac{1}{2}$, $u_3 = 2$, $u_4 = -1$

$u_4 = u_1$ so the sequence repeats with period 3

(b) $25 = 3 \times 8 + 1$, so $u_{25} = u_1 = -1$

Sum of one cycle $= \frac{3}{2}$, so the sum $= 8 \times \frac{3}{2} - 1 = 11$

Answer: period 3; $u_{25} = -1$; sum = 11

9 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $12\cos\theta + 5\sin\theta \equiv 13\cos(\theta - 22.6°)$, solve $12\cos\theta + 5\sin\theta = -1$ for $0 \le \theta \le 360°$. (1 d.p.)

$13\cos(\theta - 22.6°) = -1$, so $\cos(\theta - 22.6°) = \frac{-1}{13}$

$\theta - 22.6° = \pm 94.4°$

$\theta = 22.6° + 94.4° = 117.0°$

$\theta = 22.6° - 94.4° = -71.8°$, so $\theta = 288.2°$

Answer: θ = 117.0°, 288.2°

10 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x(x + 3)(x - 4) > 0$.

Critical values: $x = -3$, $x = 0$, $x = 4$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $-3 < x < 0$ or $x > 4$

Answer: $-3 < x < 0$ or $x > 4$