**Day 126** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Parametric to Cartesian (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations
$x = -2 + 4\cos t$, $y = -4 + 4\sin t$, for $0 \leq t < 2\pi$. Find a Cartesian equation of the curve.

$\cos t = \frac{x+2}{4}$ and $\sin t = \frac{y+4}{4}$

Use $\cos^2 t + \sin^2 t = 1$:
$(x+2)^2 + (y+4)^2 = 16$
**Answer: $(x+2)^2 + (y+4)^2 = 16$**

## 2 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $xe^y + y = 3$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (3, 0).

Differentiate with respect to x: $x\frac{dy}{dx}e^y + \frac{dy}{dx} + e^y = 0$

$\frac{dy}{dx} = -\frac{e^y}{xe^y + 1}$

At (3, 0): gradient $= -\frac{1}{4}$

**Answer: $\frac{dy}{dx} = -\frac{e^y}{xe^y + 1}$; gradient $-\frac{1}{4}$**

## 3 Standardising to compare (All boards)

Marks are normal: Maths $N(55, 10^2)$, Physics $N(40, 8^2)$.
Anna scores 67.5 in Maths and 52 in Physics. Which is her better performance, relatively?

z-score in Maths $= \frac{67.5 - 55}{10} = 1.25$
z-score in Physics $= \frac{52 - 40}{8} = 1.50$
The higher z-score is in Physics
**Answer: Physics**

## 4 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 8 and the common difference is 3.
The sum of the first n terms is 869. Find the value of n.

$S_n = \frac{n}{2}(2 \times 8 + (n-1) \times 3) = 869$

$3n^2 + 13n - 1738 = 0$
$(n - 22)(3n + 79) = 0$
n must be positive, so $n = 22$
**Answer: $n = 22$**

## 5 Double angle equations (All boards)

Solve $2\sin 2x = 3\sin x$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use the double angle formula: $4\sin x\cos x - 3\sin x = 0$
$\sin x(4\cos x - 3) = 0$
$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$
$\cos x = \frac{3}{4}$: $x = 41.4°, 318.6°$
**Answer: $x = 0.0°, 41.4°, 180.0°, 318.6°, 360.0°$**

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## 6 Binomial: smallest n (All boards)

Given that each component fails independently with probability 0.4, find the least number of components needed for the probability of at least one success to exceed 0.99.

P(at least one) = $1 - 0.6^n > 0.99$

$0.6^n < 0.01$

$n > \frac{\log 0.01}{\log 0.6} = 9.015$

**Answer: n = 10**

## 7 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 20°C: $\frac{dT}{dt} = -k(T - 20)$. Initially T = 90 and after 10 minutes T = 50.

Find k to 3 s.f. and the time to cool to 30°C, to 3 s.f.

$\frac{1}{T - 20}\,dT = -k\,dt$, so $\ln(T - 20) = -kt + c$, i.e. $T = 20 + 70e^{-kt}$

t = 10: $50 = 20 + 70e^{-10k}$, so $e^{-10k} = \frac{3}{7}$

$k = \frac{1}{10} \ln \frac{70}{30} = 0.08473$

$30 = 20 + 70e^{-kt}$: $t = \frac{1}{k} \ln \frac{70}{10} = 22.97$

**Answer: k = 0.0847, t = 23.0 minutes**

## 8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{3}$ is irrational, prove by contradiction that $3 + 6\sqrt{3}$ is irrational.

Assume $3 + 6\sqrt{3} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $6\sqrt{3} = \frac{r}{s} - 3 = \frac{r - 3s}{s}$, so $\sqrt{3} = \frac{r - 3s}{6s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{3}$ being irrational, so $3 + 6\sqrt{3}$ is irrational.

**Answer: $3 + 6\sqrt{3}$ is irrational**

## 9 Horizontal projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown horizontally at 13 m $s^{-1}$ from the top of a cliff 11.025 m above level ground.

Take g = 9.8 m $s^{-2}$.

Find the time of flight and the horizontal distance from the base where it lands.

Vertical: $11.025 = \frac{1}{2} \times 9.8 \times t^2$, $t^2 = 2.25$

t = 1.5 s

Horizontal: $x = 13 \times 1.5$

**Answer: 1.5 s, 19.5 m**

## 10 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 4 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 1 kg hanging freely.

Take g = 9.8 m $s^{-2}$.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $1g - T = 1a$

Table: $T = 4a$

$1g = 5a$, so $a = 1.96$

$T = 4 \times 1.96$

**Answer: a = 1.96 m $s^{-2}$; T = 7.84 N**