Day 125 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Boys: 50 − 25 = 25. Girls: 50 − 35 = 15
(b) Medians: 40 (Boys) and 45 (Girls), so Girls has the higher median
IQRs: Boys has the larger IQR, so its scores are more spread out
Answer: (a) 25 and 15 (b) Girls has the higher median; Boys is more spread out (larger IQR), Girls more consistent

2 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 6 cm and angle 1.4 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 6 \times 1.4 = 8.4$ cm
Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times 1.4 = 25.2$ cm^2
Answer: (a) 8.4 cm (b) 25.2 cm^2

3 Rough horizontal surface: finding μ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 19.8 N. It accelerates at 2 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the coefficient of friction between the block and the ground.

R = mg = 5 × 9.8 = 49
Resultant: 19.8 − F = 5 × 2, so F = 9.8
F = μR: μ = 9.8 ÷ 49
Answer: 0.2

4 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate y = 2 cosec 3x with respect to x,
and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule
$\frac{dy}{dx} = -6 \cot 3x \operatorname{cosec} 3x$
At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = -6\sqrt{2}$
Answer: $\frac{dy}{dx} = -6 \cot 3x \operatorname{cosec} 3x$; gradient $-6\sqrt{2}$

5 Unknown force from acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $(-\mathbf{i} + \mathbf{j})$ N and $\mathbf{P}$ N only. Its acceleration is $(-3\mathbf{i} + \mathbf{j})$ m s^{-2}.
Find $\mathbf{P}$, and its magnitude to 3 significant figures.

F = ma: $(-\mathbf{i} + \mathbf{j}) + \mathbf{P} = 5(-3\mathbf{i} + \mathbf{j}) = (-15\mathbf{i} + 5\mathbf{j})$
$\mathbf{P} = -14\mathbf{i} + 4\mathbf{j}$
$|\mathbf{P}| = \sqrt{(-14^2 + 4^2)}$
Answer: $\mathbf{P} = (-14\mathbf{i} + 4\mathbf{j})$ N; $|\mathbf{P}| = 14.6$ N

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6 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve C has equation $y = x^2 + 2x - 4$.

C is reflected in the y-axis and then translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

Find its new equation.

Reflection in the y-axis: replace x by $-x$.

When reflected in the y-axis: $y = x^2 - 2x - 4$

Translation: replace x by $(x - h)$ and add k to y.

When translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$: $y = x^2 - 8x + 9$

Answer: $y = x^2 - 8x + 9$

7 Finding the mean or standard deviation (All boards)

A journey takes X minutes, normal with s.d. 8.
Given $P(X < 88.2) = 0.95$, find the mean (3 s.f.).

$P(X < 88.2) = 0.95$, so $z = 1.645$

$\frac{88.2 - \mu}{8} = 1.645$

$\mu = 88.2 - 1.645 \times 8 = 75.0$

Answer: $\mu = 75.0$

8 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = \ln x$ to find the exact value of

$\int_1^e \frac{\ln x}{x}\,dx$.

$\frac{du}{dx} = \frac{1}{x}$, so $\frac{1}{x}\,dx = du$

Limits: $x = 1$ gives $u = 0$, $x = e$ gives $u = 1$

$\int_0^1 u^1\,du = [\frac{u^2}{2}]_0^1$

$= \frac{1}{2}$

Answer: $\frac{1}{2}$

9 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -x$. The point P is $(-2, 4)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l $= -1$, so the perpendicular has gradient 1

$y - 4 = (x + 2)$, giving $x - y + 6 = 0$

(b) Solve $y = -x$ with $x - y + 6 = 0$: $F = (-3, 3)$

$PF^2 = 1^2 + 1^2 = 2$

Answer: (a) $x - y + 6 = 0$ (b) F$(-3, 3)$, PF $= \sqrt{2}$

10 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x - 4y = k$ is a tangent to the circle
$(x - 1)^2 + (y - 1)^2 = 25$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to $(3, -4)$.

A unit step in this direction is $(3, -4)/5$, so the points of contact are

$(1, 1) \pm 5 \times (3, -4)/5 = (1, 1) \pm (3, -4)$

$T = (4, -3)$ or $(-2, 5)$, and $k = 3x - 4y$ gives:

$k = 24$ or $k = -26$

Answer: $k = 24$ or $k = -26$