Day 124 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A remote-control car moves with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(3^2 + -4^2)} = 5$

$\tan\theta = 4 \div 3$

Answer: 5 m s^{-1}; 53.1°

2 Quartiles and outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Ordered data, texts sent in a day by 7 students:

13, 23, 31, 34, 35, 39, 39.

Find Q_1, Q_3, the IQR and any outliers (more than 1.5 × IQR beyond a quartile).

$Q_1 = 23$, $Q_3 = 39$, IQR = 16

Lower limit = $23 - 1.5 \times 16 = -1$

Upper limit = $39 + 1.5 \times 16 = 63$

No value lies outside these limits.

Answer: $Q_1 = 23$, $Q_3 = 39$, IQR = 16; no outliers

3 Derivatives of tan, sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Differentiate $y = 3\tan 3x$ with respect to x,

and find the exact gradient when $x = \frac{\pi}{12}$.

Use the standard results for tan, sec, cosec and cot with the chain rule

$\frac{dy}{dx} = 9\sec^2 3x$

At $x = \frac{\pi}{12}$: $\frac{dy}{dx} = 18$

Answer: $\frac{dy}{dx} = 9\sec^2 3x$; gradient 18

4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{3 - \sqrt{3}}{5 + 3\sqrt{3}}$ in the form $p + q\sqrt{3}$, where p and q are integers.

Multiply top and bottom by the conjugate $(5 - 3\sqrt{3})$:

Denominator: $(5 + 3\sqrt{3})(5 - 3\sqrt{3}) = 5^2 - 3^2 \times 3 = -2$

Numerator: $(3 - \sqrt{3})(5 - 3\sqrt{3}) = 24 - 14\sqrt{3}$

So the fraction = $\frac{24 - 14\sqrt{3}}{-2} = -12 + 7\sqrt{3}$

Answer: $-12 + 7\sqrt{3}$

5 Conditional probability formula (All boards)

P(B) = 0.25, P(A | B) = 0.6 and P(A) = 0.5.

(a) Find $P(A \cap B)$.

(b) Find P(B | A).

(c) Find $P(A \cup B)$.

(a) $P(A \cap B) = P(A \mid B) \times P(B) = 0.6 \times 0.25 = 0.15$

(b) $P(B \mid A) = \frac{0.15}{0.5} = 0.3$

(c) $P(A \cup B) = 0.5 + 0.25 - 0.15 = 0.6$

Answer: (a) 0.15 (b) 0.3 (c) 0.6

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.
(a) Find the coordinates of the local maximum point of y = |f(x)|.
(b) How many solutions has |f(x)| = 3?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:
(2, −1) becomes (2, 1)
(b) The maximum of the middle part is 1. Compare k = 3 with 1:
k > 1 gives 2 intersections with y = k.
Answer: (a) (2, 1) (b) 2

7 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass M kg of a substance satisfies $\frac{dM}{dt} = -0.05M$, where t is in years.
Initially M = 100. Find M when t = 10, and the half-life, each to 3 s.f.

$\frac{1}{M}\,dM = -0.05\,dt$, so $\ln M = -0.05t + c$, giving $M = 100e^{-0.05t}$

t = 10: $M = 100\,e^{-0.5} = 60.65$

Half-life: $e^{-0.05t} = \frac{1}{2}$, so $t = \frac{\ln 2}{0.05} = 13.86$

Answer: M = 60.7 kg, half-life 13.9 years

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{3}$ is irrational, prove by contradiction that $1 + 4\sqrt{3}$ is irrational.

Assume $1 + 4\sqrt{3} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $4\sqrt{3} = \frac{r}{s} - 1 = \frac{r - 1s}{s}$, so $\sqrt{3} = \frac{r - 1s}{4s}$.

The right-hand side is a ratio of integers, so it is rational.
This contradicts $\sqrt{3}$ being irrational, so $1 + 4\sqrt{3}$ is irrational.
Answer: $1 + 4\sqrt{3}$ is irrational

9 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 90 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 20 to t = 70.
The floors are 840 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (90 + 50) × V = 840
140 V = 1680 so V = 12
Deceleration = 12 ÷ 20
Answer: V = 12; deceleration 0.6 m s^{-2}

10 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x - 3\sec x - 7 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x - 3\sec x - 9 = 0$

$(2\sec x + 3)(\sec x - 3) = 0$, so $\sec x = -\frac{3}{2}$ or 3

$\sec x = -\frac{3}{2}$, so $\cos x = -\frac{2}{3}$: x = 131.8°, 228.2°

$\sec x = 3$, so $\cos x = \frac{1}{3}$: x = 70.5°, 289.5°

Answer: x = 70.5°, 131.8°, 228.2°, 289.5°