**Day 123** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Overtaking problem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A police car starts from rest and accelerates at 2 m $s^{-2}$. At that moment it is passed by a speeding van travelling at constant speed 17 m $s^{-1}$.
Find how long it takes to catch up, and the distance travelled.

Same distance: ½ × 2 × $t^2$ = 17 t
t(1 t − 17) = 0, t ≠ 0 so t = 17 ÷ 1
Distance = 17 × 17
**Answer: 17 s, 289 m**

## 2 Trig equations in radians (All boards)

Solve tan x = 0.5 for 0 ≤ x ≤ 2π,
giving answers to 3 significant figures.

tan θ = 0.5 for 0 ≤ x ≤ 2π
Calculator: x = 0.4636, then use the symmetry of the graph
x = 0.4636, 3.605
**Answer: x = 0.464, 3.61**

## 3 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (3, 1), (−1, 4) and (2, 8).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{-3}{4} = -\frac{3}{4}$

Gradient BC = $\frac{4}{3}$

$(-\frac{3}{4}) \times \frac{4}{3} = -1$, so AB and BC are perpendicular.

(b) AB = 5 and BC = 5

Area = $\frac{1}{2} \times AB \times BC = \frac{25}{2}$

**Answer: (b) area = $\frac{25}{2}$**

## 4 Constant acceleration, two stages of working (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line with constant acceleration increases its speed from 9 m $s^{-1}$ to 13.5 m $s^{-1}$ while covering 101.25 m.
Find (a) the acceleration, (b) the time taken.

(a) $v^2 = u^2 + 2as$: 182.25 = 81 + 202.5 a, so a = 0.5
(b) v = u + at: 13.5 = 9 + 0.5 t
t = 4.5 ÷ 0.5 = 9
**Answer: (a) 0.5 m $s^{-2}$ (b) 9 s**

## 5 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve (x − 3)(x − 2)(x − 5) > 0.

Critical values: x = 2, x = 3, x = 5
The cubic has a positive $x^3$ term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: 2 < x < 3 or x > 5
**Answer: 2 < x < 3 or x > 5**

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## 6 Rates of change (connected rates) (All boards)

The radius of a circular oil slick increases at 1.5 cm per second.
Find the rate at which its area is increasing when r = 4 cm, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi r \times 1.5$

When r = 4: $\frac{dA}{dt} = 12\pi$

**Answer: $12\pi$ cm$^2$/s**

## 7 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 48) = 0.1587 and P(X > 72) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -0.9998$ and $z_2 = 0.9998$
$48 = \mu + (-0.9998)\sigma$ and $72 = \mu + 0.9998\sigma$
Subtract: $24 = 2.000\,\sigma$, so $\sigma = 12.0$
$\mu = 48 - (-0.9998)(12.00) = 60.0$
**Answer: $\mu = 60.0$, $\sigma = 12.0$**

## 8 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $7\cos\theta + 24\sin\theta \equiv 25\cos(\theta - 73.7°)$,
find the greatest value of $\frac{1}{(27 + 7\cos\theta + 24\sin\theta)}$.

Denominator is smallest when $25\cos(\theta - 73.7°) = -25$
Smallest denominator = 27 − 25 = 2
Greatest value = $\frac{1}{2}$

**Answer: $\frac{1}{2}$**

## 9 Quadratics in a^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the equation $5^{2x} - 120\times5^x - 625 = 0$.

Let $y = 5^x$. Then $5^{2x} = y^2$, so $y^2 - 120y - 625 = 0$
$(y - 125)(y + 5) = 0$
$y = 125$ or $y = -5$
$5^x > 0$ for all x, so $y = -5$ is rejected. $5^x = 125$ gives $x = 3$.
**Answer: x = 3**

## 10 Binomial one-tailed test (All boards)

A manufacturer claims that a proportion 0.5 of its sweets are red. A sample of 25 sweets has 14 red.
Test, at the 5% level, whether the true proportion is higher than claimed.

$H_0$: p = 0.5, $H_1$: p > 0.5. Let X ~ B(25, 0.5) under $H_0$
$P(X \geq 14) = 1 - P(X \leq 13) = 0.3450$
0.3450 > 0.05, so do not reject $H_0$
There is insufficient evidence that the proportion is higher than claimed
**Answer: do not reject $H_0$: insufficient evidence that the proportion is higher**