Day 122 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(-7\mathbf{i} + 24\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors east and north. Find its speed and the acute angle its direction makes with $\mathbf{i}$, to 1 d.p.

Speed = $\sqrt{(-7^2 + 24^2)} = 25$

$\tan\theta = 24 \div 7$

Answer: 25 m s^{-1}; 73.7°

2 Evaluating fractional indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $4^{\frac{1}{2}}$.

$4 = 2^2$, so $4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^1$

$= 2$

Answer: 2

3 Solving exponential equations using logarithms (All boards)

Solve $2^x = 48$, giving your answer to 3 s.f.

Take logs: $x \ln 2 = \ln 48$

$x = \dfrac{\ln 48}{\ln 2} = 5.585$

Answer: x = 5.58

4 Inverse normal (All boards)

A journey takes X minutes, normal with mean 100, s.d. 10. Find k (3 s.f.): (a) P(X < k) = 0.05, (b) P(X > k) = 0.01.

(a) $z = -1.645$, so $k = 100 + (-1.645) \times 10 = 83.6$

(b) P(X < k) = 0.99, $z = 2.326$, so $k = 100 + 2.326 \times 10 = 123$

Answer: (a) 83.6 (b) 123

5 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 2x^2 - x + k$. Given that $(x - 2)$ is a factor of f(x), (a) find the value of k, (b) factorise f(x) completely.

$f(2) = 0$: $8 - 8 - 2 + k = 0$

$k = 2$

$f(x) = (x - 2)(x^2 - 1)$

$= (x - 2)(x - 1)(x + 1)$

Answer: (a) k = 2 (b) f(x) = (x − 2)(x − 1)(x + 1)

6 Segment of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The shaded region is the segment cut off by the chord AB of a circle with centre O and radius 8 cm. Angle AOB = 0.9 radians. Find the area of the segment, to 3 s.f.

Sector area = $\frac{1}{2} \times 8^2 \times 0.9 = 28.8$

Triangle area = $\frac{1}{2} \times 8^2 \times \sin 0.9 = 25.07$

Segment = sector − triangle = 3.734 cm^2

Answer: 3.73 cm^2

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7 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 + 4x)(1 - 3x)^7$.

$(1 - 3x)^7$ has x^1 coefficient -21 and x^2 coefficient 189
x^2 term of the product = $1 \times 189 + 4 \times (-21)$
= 105
Answer: 105

8 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}3\\0\\3\end{pmatrix}$ and $\begin{pmatrix}5\\10\\14\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}2\\10\\11\end{pmatrix}$

$|\text{AB}| = \sqrt{2^2 + 10^2 + 11^2} = \sqrt{225} = 15$

Answer: AB = $\begin{pmatrix}2\\10\\11\end{pmatrix}$, distance 15

9 Velocity parallel to a direction (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves so that at time t s its velocity is $\mathbf{v} = (3t^2 + 2)\mathbf{i} + (2t - 2)\mathbf{j}$ m s^{-1}.
Find the time when the particle is moving in the direction of $\mathbf{i}$, and its speed then.

Parallel to $\mathbf{i}$: $\mathbf{j}$ component = 0
$2t - 2 = 0$, so $t = 1$
$\mathbf{v} = (3 \times 1^2 + 2)\mathbf{i} = 5\mathbf{i}$
Answer: t = 1 s, speed 5 m s^{-1}

10 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(150, 0.5). Use a normal approximation to find $P(X \leq 76)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(75.00, 37.50)$, so $\sigma = \sqrt{37.50} = 6.124$
Use a continuity correction.
$P(X \leq 76) \approx P(Y < 76.5) = P(Z < 0.2449) = 0.597$
Answer: 0.597