Day 121 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Product rule (All boards)

Given that $y = e^{2x} \cos x$,

find $\frac{dy}{dx}$, and its value when $x = \frac{\pi}{2}$.

Product rule with $u = e^{2x}$, $v = \cos x$

$u' = 2e^{2x}$, $v' = -\sin x$

$\frac{dy}{dx} = u'v + uv' = (2\cos x - \sin x)e^{2x}$

When $x = \frac{\pi}{2}$: $\frac{dy}{dx} = -e^{\pi}$

Answer: $\frac{dy}{dx} = (2\cos x - \sin x)e^{2x}$; value $-e^{\pi}$

2 Modulus inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 7| < 5$.

$|2x - 7| < 5$ means $-5 < 2x - 7 < 5$

Subtract -7 and divide by 2: $1 < x < 6$

Answer: $1 < x < 6$

3 Poisson distribution (CIE)

X ~ Po(2.5), modelling the number of flaws per metre of cloth.

Find, to 3 significant figures,

(a) $P(X \geq 4)$

(b) $P(X \leq 4)$

(a) $P(X \geq 4) = 1 - P(X \leq 3) = 0.242$

(b) $P(X \leq 4) = P(X = 0) + ... + P(X = 4) = 0.891$

Answer: (a) 0.242 (b) 0.891

4 Integrals of the form f′(x)/f(x) (All boards)

Find the exact value of

$\int_0^{\ln 3} \frac{e^x}{e^x + 2}\, dx$,

giving your answer in the form a ln b.

The numerator is a multiple of the derivative of the denominator

$\int \frac{e^x}{e^x + 2}\, dx = \ln(e^x + 2)$

$= [\ln(e^x + 2)]_0^{\ln 3}$

$= \ln(\frac{5}{3})$

Answer: $\ln(\frac{5}{3})$

5 Pulling at an angle: constant speed on rough surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A trunk of mass 12 kg is dragged at constant speed along rough horizontal ground by a rope inclined at angle α above the horizontal, where $\cos\alpha = \frac{12}{13}$.

The coefficient of friction is 0.2. Take $g = 9.8$ m s^{-2}. Find the tension in the rope.

Vertical: $R + T\sin\alpha = mg$, so $R = 117.6 - \frac{5}{13}T$

Horizontal: $T\cos\alpha = \mu R$: $\frac{12}{13}T = 0.2(117.6 - \frac{5}{13}T)$

$T(\cos\alpha + \mu\sin\alpha) = \mu mg$: $T \times 1 = 0.2 \times 12 \times 9.8$ (since $\cos\alpha + \mu\sin\alpha = 1$)

Answer: 23.52 N

6 Continuous random variable: pdf (CIE)

The continuous random variable X has pdf $f(x) = kx^3$ for $0 \le x \le 3$, and 0 otherwise.
(a) Find the value of k.
(b) Find P(X > 1). (c) Find E(X).

(a) $\int f(x)\,dx = 1$: $k[x^4/4]$ from 0 to 3 $= k \times \frac{3^4}{4} = 1$, so $k = \frac{4}{81}$

(b) $P(X > 1) = \int_1^3 \frac{4}{81} x^3\,dx = 1 - \frac{1^4}{3^4} = \frac{80}{81}$

(c) $E(X) = \int_0^3 x\,f(x)\,dx = \frac{4}{81} \times \frac{3^5}{5} = \frac{12}{5}$

Answer: (a) $k = \frac{4}{81}$ (b) $\frac{80}{81}$ (c) $\frac{12}{5}$

7 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x + 3)(x + 2)(x - 5) < 0$.

Critical values: $x = -3$, $x = -2$, $x = 5$

The cubic has a positive x^3 term, so the graph goes from bottom left to top right and crosses the axis at each critical value.
Reading from the sketch: $x < -3$ or $-2 < x < 5$

Answer: $x < -3$ or $-2 < x < 5$

8 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (9 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 8.75 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (9 - 5)g \div (9 + 5) = 2.8$
$v^2 = 0 + 2 \times 2.8 \times 8.75 = 49$
$v = 7$

Answer: 7 m s^{-1}

9 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of cot 45°,
giving your answer with a rational denominator.

$\cot 45° = \frac{1}{\tan 45°}$

$\tan 45° = 1$, so $\cot 45° = \frac{1}{1}$

Answer: 1

10 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = x - 15$ meets the circle $x^2 + (y + 2)^2 = 125$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = x - 15$: $x^2 + (x - 13)^2 = 125$
$x^2 - 13x + 22 = 0$
$(x - 11)(x - 2) = 0$, so $x = 2$ or $x = 11$
$y = x - 15$: A = (2, −13), B = (11, −4)
$AB^2 = 9^2 + 9^2 = 162$

Answer: A(2, −13), B(11, −4), AB = $9\sqrt{2}$