Day 120 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from ground level with initial velocity components 31 m s^{-1} horizontally and 19.6 m s^{-1} vertically upwards.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 19.6^2 - 2 \times 9.8 \times H$, $H = 384.16 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 19.6 m (b) 31 m s^{-1}

2 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -4 \\ -4 \\ 7 \end{pmatrix}$.

Find |a| and a unit vector in the direction of a.

$|\mathbf{a}| = \sqrt{4^2 + 4^2 + 7^2} = \sqrt{81} = 9$

Unit vector $= \frac{1}{9}\mathbf{a}$

Answer: |a| = 9, unit vector $\frac{1}{9}\begin{pmatrix} -4 \\ -4 \\ 7 \end{pmatrix}$

3 Constant velocity: position vector (AQA, Edexcel, OCR, MEI, Eduqas)

A radio-controlled car moves with constant velocity $(-2\mathbf{i} - 2\mathbf{j})$ m s^{-1}. At time t = 0 its position vector relative to a fixed origin O is $(-7\mathbf{i} + 4\mathbf{j})$ m.

Find (a) its position vector when t = 6, (b) the value of t when it crosses the line through O in the direction of **i**.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-7\mathbf{i} + 4\mathbf{j}) + 6(-2\mathbf{i} - 2\mathbf{j}) = (-19\mathbf{i} - 8\mathbf{j})$

(b) **j** component zero: $4 - 2t = 0$, so $t = 2$

Answer: (a) (−19i − 8j) m (b) t = 2 s

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: maximum relative humidity (%) at Hurn has mean 88, sd 7.

Outliers are more than 3 sd from the mean.

Are the values 111 and 69 outliers?

Limits: $88 \pm 3 \times 7$, so from 67 to 109

111 is outside the limits, so it is an outlier

69 is inside the limits, so it is not an outlier

Answer: 111: outlier; 69: not an outlier

5 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + xy + 4x + 3y^2 - 2y = 45$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (3, −3).

Differentiate with respect to x: $x\frac{dy}{dx} + 2x + 6y\frac{dy}{dx} + y + 4 - 2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x - y - 4}{x + 6y - 2}$

At (3, −3): gradient $= \frac{7}{17}$

Answer: $\frac{dy}{dx} = \frac{-2x - y - 4}{x + 6y - 2}$**; gradient** $\frac{7}{17}$

6 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{216}$ for $0 \leq x \leq 6$, and 0 otherwise.

(a) Find $P(X > 4)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{216}$

(a) $P(X > 4) = 1 - F(4) = \frac{19}{27}$

(b) $F(m) = 0.5$: $\frac{m^3}{216} = \frac{1}{2}$

$m = 4.76$

Answer: (a) $\frac{19}{27}$ (b) 4.76

7 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $4x^2 + 10x < -4$.

Rearrange: $4x^2 + 10x + 4 < 0$
Factorise: $2(x + 2)(2x + 1) < 0$

Critical values: $x = -2$ and $x = -\frac{1}{2}$

Sketch the ∪-shaped parabola: $-2 < x < -\frac{1}{2}$

Answer: $-2 < x < -\frac{1}{2}$

8 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}8\\3\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\0\\3\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}4\\0\\-5\end{pmatrix} + \mu\begin{pmatrix}0\\1\\1\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $8 + 2\lambda = 4 + 0\mu$
j: $3 + 0\lambda = 0 + 1\mu$
Solving i and j: $\lambda = -2$, $\mu = 3$
k check: $4 + 3(-2) = -2$ and $-5 + 1(3) = -2$, equal

Point: $\begin{pmatrix}4\\3\\-2\end{pmatrix}$

Answer: $\begin{pmatrix}4\\3\\-2\end{pmatrix}$

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 10$.

Assume $x^2 - y^2 = 10$, so $(x - y)(x + y) = 10$.
$x - y$ and $x + y$ have the same parity (their sum $2x$ is even).
If both are odd, the product is odd; if both are even, the product is a multiple of 4.
But 10 is even and is not a multiple of 4 ($10 = 4 \times 2 + 2$), a contradiction.
So no such integers exist.
Answer: no integer solutions (product is odd or a multiple of 4)

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10 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{-3x + 2}{x - 3}$ for $x \in \mathbb{R}$, $x \neq 3$.

(a) Find $f^{-1}(x)$.

(b) State the domain of f^{-1}.

Let $y = \frac{-3x + 2}{x - 3}$. Then $y(x - 3) = -3x + 2$

$xy - 3y = -3x + 2$, so $x(y + 3) = 3y + 2$

$f^{-1}(x) = \frac{3x + 2}{x + 3}$

f never takes the value -3, so the domain of f^{-1} is $x \neq -3$.

Answer: (a) $f^{-1}(x) = \frac{3x + 2}{x + 3}$ (b) $x \neq -3$