Day 12 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 48 students.
Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 48$, $\Sigma fx = 195$, $\Sigma fx^2 = 925$

Mean = $\frac{195}{48} = 4.06$

Variance = $\frac{925}{48} - (4.0625)^2 = 2.767$

Standard deviation = 1.66

Answer: mean 4.06, standard deviation 1.66

2 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 70 kg stands on weighing scales in a lift. The lift decelerates at 0.25 m s^{-2} while moving upwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 0.25
Taking up as positive: $R - 70 \times 9.8 = -70 \times 0.25$
$R = 70(9.8 - 0.25)$

Answer: 668.5 N

3 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = kx^2$ for x = 1, 2, ..., 4.
(a) Find k.
(b) Find E(X) and Var(X).

(a) $k(1 + 4 + 9 + 16) = 1$, so $30k = 1$ and $k = \frac{1}{30}$

(b) $E(X) = 1 \times \frac{1}{30} + 2 \times \frac{2}{15} + 3 \times \frac{3}{10} + 4 \times \frac{8}{15} = \frac{10}{3}$

$E(X^2) = \frac{59}{5}$, so $Var(X) = \frac{59}{5} - (\frac{10}{3})^2 = \frac{31}{45}$

Answer: (a) $k = \frac{1}{30}$ (b) $E(X) = \frac{10}{3}$, $Var(X) = \frac{31}{45}$

4 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A person of mass 80 kg stands on the floor of a lift. The lift accelerates downwards at 0.5 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the normal reaction between the person and the floor.

Resolve vertically (up positive): $R - 80 \times 9.8 = -80 \times 0.5$
$R = 80(9.8 - 0.5)$

Answer: 744 N

5 Right-angled triangle on a grid (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C are (−1, 5), (2, 3) and (6, 9).
(a) Show that angle ABC = 90°.
(b) Find the area of triangle ABC.

(a) Gradient AB = $\frac{2}{-3} = -\frac{2}{3}$

Gradient BC = $\frac{6}{4} = \frac{3}{2}$

$(-\frac{2}{3}) \times \frac{3}{2} = -1$, so AB and BC are perpendicular.

(b) AB = $\sqrt{13}$ and BC = $2\sqrt{13}$

Area = $\frac{1}{2} \times AB \times BC = 13$

Answer: (b) area = 13

6 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ab^x$. A graph of ln y against x is a straight line with gradient ln 5 that passes through (0, ln 2).

Find the values of a and b.

ln y = ln a + x ln b

gradient = ln b = ln 5, so b = 5

intercept ln a = ln 2, so a = 2

Answer: a = 2, b = 5

7 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{-2x^2 + 26x + 64}{(x - 5)(x + 1)^2}$ in partial fractions.

$\frac{-2x^2 + 26x + 64}{(x - 5)(x + 1)^2} \equiv \frac{A}{(x - 5)} + \frac{B}{(x + 1)} + \frac{C}{(x + 1)^2}$

$-2x^2 + 26x + 64 \equiv A(x + 1)^2 + B(x - 5)(x + 1) + C(x - 5)$

x = 5 gives A = 4; x = −1 gives C = −6

Coefficients of x^2: −2 = A + B, so B = −6

Answer: $\frac{4}{(x - 5)} - \frac{6}{(x + 1)} - \frac{6}{(x + 1)^2}$

8 Optimisation: open box and tank (All boards)

A closed cylinder has radius r cm and volume 16π cm^3.

Show that its surface area is $S = 2\pi r^2 + \frac{32\pi}{r}$, and find the minimum S.

Height $h = \frac{16}{r^2}$, so $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + \frac{32\pi}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{32\pi}{r^2} = 0$, so $r^3 = 8$ and r = 2

$\frac{d^2S}{dr^2} = 4\pi + \frac{64\pi}{r^3} > 0$, so minimum

$S = 2\pi \times 4 + \frac{32\pi}{2} = 24\pi$ cm^2

Answer: S = 24π cm^2

9 Periodic sequences (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{1 - u_n}$, $u_1 = 3$.

(a) Show that the sequence is periodic, stating the period.

(b) Find u_{20} and $\sum_{r=1}^{20} u_r$.

(a) $u_2 = -\frac{1}{2}$, $u_3 = \frac{2}{3}$, $u_4 = 3$

$u_4 = u_1$ so the sequence repeats with period 3

(b) 20 = 3 × 6 + 2, so $u_{20} = u_2 = -\frac{1}{2}$

Sum of one cycle = $\frac{19}{6}$, so the sum = $6 \times \frac{19}{6} + \frac{5}{2} = \frac{43}{2}$

Answer: period 3; $u_{20} = -\frac{1}{2}$; sum = $\frac{43}{2}$

10 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix}0\\5\\8\end{pmatrix} + \lambda\begin{pmatrix}-2\\3\\3\end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix}-2\\-1\\8\end{pmatrix} + \mu\begin{pmatrix}2\\0\\-2\end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $0 + (-2)\lambda = -2 + 2\mu$

j: $5 + 3\lambda = -1 + 0\mu$

Solving i and j: $\lambda = -2$, $\mu = 3$

k check: $8 + 3(-2) = 2$ and $8 + (-2)(3) = 2$, equal

Point: $\begin{pmatrix}4\\-1\\2\end{pmatrix}$

Answer: $\begin{pmatrix}\mathbf{4}\\\mathbf{-1}\\\mathbf{2}\end{pmatrix}$