Day 119 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Series in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A charity raises £250 in week 1. In each following week it raises £10 more than in the previous week.
(a) Find the amount raised in week 12.
(b) Find the total for the first 26 weeks.

(a) Arithmetic sequence with a = 250, d = 10
$u_{12} = 250 + 11 \times 10 = £360$

(b) $S_{26} = \frac{26}{2}(2 \times 250 + 25 \times 10) = £9{,}750$

Answer: (a) £360 (b) £9,750

2 Solving exponential equations using logarithms (All boards)

Solve $4^x = 52$, giving your answer to 3 s.f.

Take logs: $x \ln 4 = \ln 52$

$x = \frac{\ln 52}{\ln 4} = 2.850$

Answer: x = 2.85

3 Power, driving force and resistance (CIE)

A car of mass 1500 kg moves along a horizontal road. The engine works at a constant rate of 24 kW. The resistance to motion is constant at 600 N.
Find the acceleration when the speed is 10 m s^{-1}.

P = Fv: F = 24000 ÷ 10 = 2400 N
F − R = ma: 2400 − 600 = 1500 a

Answer: 1.2 m s^{-2}

4 Expectation and variance of aX + b (AQA, OCR, MEI, Eduqas, CIE)

The random variable X has E(X) = 4 and Var(X) = 4.
Find (a) E(4X − 4),
(b) Var(4X − 4), (c) $E(X^2)$.

(a) E(aX + b) = aE(X) + b = 4 × 4 − 4 = 12
(b) $Var(aX + b) = a^2Var(X) = 16 \times 4 = 64$
(c) $Var(X) = E(X^2) - [E(X)]^2$, so $E(X^2) = 4 + 16 = 20$
Answer: (a) 12 (b) 64 (c) 20

5 Equilibrium of three forces (components) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is in equilibrium under three forces: P N at angle θ above the positive x-axis, a force of 12 N in the negative x-direction and a force of Q N in the negative y-direction.
$\cos\theta = \frac{3}{5}$. Find P and Q.

Resolve horizontally: $P\cos\theta = 12$, so $P \times \frac{3}{5} = 12$

P = 20

Resolve vertically: $Q = P\sin\theta = 20 \times \frac{4}{5}$

Answer: P = 20 N; Q = 16 N

6 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $3\cos^2 x + 1 - 4\cos x = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(3\cos x - 1)(\cos x - 1) = 0$, so $\cos x = \frac{1}{3}$ or $\cos x = 1$

$\cos x = \frac{1}{3}$: x = 70.5°, 289.5°

$\cos x = 1$: x = 0.0°, 360.0°

Answer: x = 0.0°, 70.5°, 289.5°, 360.0°

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7 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x + 1} + 4$.
(a) Find the largest possible domain of f.
(b) Find the range of f.

The expression under the root must be non-negative: $x + 1 \geq 0$
$x \geq -1$ gives $x \geq -1$
$\sqrt{x + 1} \geq 0$, so $f(x) \geq 4$
Answer: (a) $x \geq -1$ (b) $f(x) \geq 4$

8 Locating roots: change of sign (All boards)

The function $f(x) = \ln x + x - 4$.
Show that $f(x) = 0$ has a root between $x = 2.5$ and $x = 3$.

$f(2.5) = -0.584$
$f(3) = 0.0986$
f is continuous and changes sign, so there is a root in the interval
Answer: f(2.5) < 0 and f(3) > 0: sign change

9 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = f(x)$. A and B are the turning points. Find the turning points of:

(a) $y = 2f(x)$ (b) $y = f(\frac{x}{2})$

(a) $y = 2f(x)$: A(0, 3) maps to (0, 6), B(4, −3) maps to (4, −6)

(b) $y = f(\frac{x}{2})$: A(0, 3) maps to (0, 3), B(4, −3) maps to (8, −3)

Answer: (a) (0, 6), (4, −6) (b) (0, 3), (8, −3)

10 Grouped data: mean and median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises 42 measurements of m.
(a) Estimate the mean.
(b) Estimate the median using linear interpolation.

(a) Using mid-points: $\Sigma fx = 2100$, so mean = 2100 ÷ 42 = 50.0

(b) $\frac{n}{2} = 21$. Cumulative frequency before the median class = 20

Median class 40 to 60. Median = $40 + \frac{1}{3} \times 20 = 46.7$

Answer: mean ≈ 50.0, median ≈ 46.7