Day 118 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t + 2$, $y = 2t^2 + 4t - 4$.
The line $y = 2x - 4$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $2t^2 + 4t - 4 = 2t$
$2t^2 + 2t - 4 = 0$, so $2(t - 1)(t + 2) = 0$
$t = -2$ or $t = 1$, giving the points $(0, -4)$ and $(3, 2)$
Answer: (0, −4) and (3, 2)

2 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 40 kg stands on weighing scales in a lift. The lift decelerates at 0.25 m s^{-2} while moving downwards.
Take g = 9.8 m s^{-2}.
Find the force exerted on the woman by the scales.

The acceleration is upwards, magnitude 0.25
Taking up as positive: $R - 40 \times 9.8 = 40 \times 0.25$
$R = 40(9.8 + 0.25)$
Answer: 402 N

3 Equation of a circle from given information (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre $(-2, -4)$ and passes through the point $(-2, 5)$.
Find an equation of the circle.

$r^2 = (-2 - (-2))^2 + (5 - (-4))^2$
$= 0 + 81 = 81$
Answer: $(x + 2)^2 + (y + 4)^2 = 81$

4 Smooth inclined plane (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A toy car slides from rest down a line of greatest slope of a smooth plane inclined at α, where $\sin\alpha = \frac{3}{5}$ to the horizontal.
Take g = 9.8 m s^{-2}.
Find its acceleration, and the distance it travels in 2 s.

Along the plane: $mg \sin\alpha = ma$, so $a = 9.8 \times 0.6 = 5.88$
$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5.88 \times 2^2$
Answer: 5.88 m s^{-2}; 11.76 m

5 Exact values of sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\tan\theta = 3$ and θ is acute,
find the exact value of $\sec\theta$.

Use $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
$\tan\theta = 3$, so $\sec^2\theta = 1 + 9 = 10$
$\sec\theta = \sqrt{10}$ (positive as θ is acute)
Answer: $\sqrt{10}$

6 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|2x - 3| = |-x + 4|$.

Either $2x - 3 = -x + 4$ or $2x - 3 = -(-x + 4)$
$2x - 3 = -x + 4$ gives $x = \frac{7}{3}$
$2x - 3 = -(-x + 4)$ gives $x = -1$
Answer: $x = -1$ and $x = \frac{7}{3}$

7 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value, in radians, of

(a) $\arccos(-\frac{1}{2})$ (b) $\arcsin(\frac{1}{2})$

(a) arccos lies in $[0, \pi]$, and $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$, so the value is $\frac{2\pi}{3}$

(b) arcsin lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and $\sin(\frac{\pi}{6}) = \frac{1}{2}$, so the value is $\frac{\pi}{6}$

Answer: (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$

8 Poisson: changing the interval (CIE)

Meteors are seen at random at an average rate of 1.5 per hour.
Find the probability that in 90 minutes there are at least 3 (3 s.f.).

Rate in 90 minutes: $\lambda = 1.5 \times \frac{90}{60} = 2.25$

$X \sim Po(2.25)$

$P(X \geq 3) = 1 - P(X \leq 2) = 0.391$

Answer: 0.391

9 Exponential quadratics and ln equations (All boards)

Solve $\ln(5x - 5) = 5$, giving your answer to 3 s.f.

$5x - 5 = e^5$

$x = \frac{e^5 + 5}{5} = 30.68$

Answer: x = 30.7

10 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{2x}{36}$ for $0 \leq x \leq 6$, and 0 otherwise.

(a) Find $P(X > 2)$.
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^2}{36}$

(a) $P(X > 2) = 1 - F(2) = \frac{8}{9}$

(b) $F(m) = 0.5$: $\frac{m^2}{36} = \frac{1}{2}$

$m = 4.24$

Answer: (a) $\frac{8}{9}$ (b) 4.24