Day 117 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 276; systematic sample of size 23.
(a) Find the sampling interval.
(b) The first item is number 7. Find the 3-th item.

(a) Interval = 276 ÷ 23 = 12
(b) 3-th item = 7 + 2 × 12 = 31
Answer: interval 12; item number 31

2 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 200 kg is raised and lowered by a vertical cable. The lift accelerates upwards at 0.25 m s^{-2}.
Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolve vertically (up positive): $T - 200 \times 9.8 = 200 \times 0.25$
$T = 200(9.8 + 0.25)$
Answer: 2010 N

3 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-2x^2 + 8x - 6 \geq 0$.

Factorise: $-2(x - 3)(x - 1) \geq 0$
Critical values: x = 1 and x = 3
Sketch the ∩-shaped parabola: $1 \leq x \leq 3$
Answer: $1 \leq x \leq 3$

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

15 pairs of height and arm span give r = 0.390.
One-tailed critical value (5%): 0.4409.
Test for positive correlation.

H_0: ρ = 0, H_1: ρ > 0
r = 0.390, critical value 0.4409; r is not beyond the critical value in the right direction
r is not in the critical region, so do not reject H_0
Conclusion: there is insufficient evidence of positive correlation between height and arm span
Answer: do not reject H_0: there is insufficient evidence of positive correlation between height and arm span

5 Area under a curve (All boards)

The region R is bounded by the x-axis, the lines x = 1
and x = e, and the curve $y = 2x + \frac{2}{x}$. Find the exact area of R.

Area = $\int_1^e (2x + \frac{2}{x})\,dx = [x^2 + 2\ln x]_1^e$
$= 1 + e^2$
Answer: $1 + e^2$

6 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve y = f(x), where f is a cubic. The curve meets the axes at the labelled points. Find f(x) in factorised form.

x-intercepts at x = −4, −1 and 1, so $y = k(x + 4)(x + 1)(x - 1)$
The curve passes through (0, 4): $k \times 4 \times 1 \times (-1) = 4$
So k = −1
Answer: $f(x) = -(x + 4)(x + 1)(x - 1)$

7 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $4\cos^2x - 3\sin x - 4 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\cos^2x = 1 - \sin^2x$: $-4\sin^2x - 3\sin x = 0$, i.e. $4\sin^2x + 3\sin x = 0$

$(4\sin x + 3)(\sin x) = 0$, so $\sin x = -\frac{3}{4}$ or 0

$\sin x = -\frac{3}{4}$: $x = 228.6°, 311.4°$

$\sin x = 0$: $x = 0.0°, 180.0°, 360.0°$

Answer: x = 0.0°, 180.0°, 228.6°, 311.4°, 360.0°

8 Partial fractions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $\frac{3x^2 - 12x + 1}{(x-3)(x+1)}$ in the form $P + \frac{Q}{(x-3)} + \frac{R}{(x+1)}$.

$3x^2 - 12x + 1 = 3(x-3)(x+1) + (-6x + 10)$, so $P = 3$

$\frac{3x^2 - 12x + 1}{(x-3)(x+1)} \equiv 3 + \frac{Q}{(x-3)} + \frac{R}{(x+1)}$

$x = 3$ gives $Q = -2$; $x = -1$ gives $R = -4$

Answer: $3 + -\frac{2}{(x-3)} - \frac{4}{(x+1)}$

9 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 3t^3 + 2t^2$ metres.
Find (a) the average velocity over the first 6 seconds, (b) the velocity at $t = 6$.

(a) $s(6) = 720$, so average velocity $= 720 \div 6$

(b) $v = ds/dt = 9t^2 + 4t$, $v(6) = 348$

Answer: (a) 120 m s^{-1} (b) 348 m s^{-1}

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{7}{25}$ and $\cos B = \frac{8}{17}$, where A and B are acute,
find the exact value of $\cos(A + B)$.

$\cos A = \frac{24}{25}$ and $\sin B = \frac{15}{17}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \frac{24}{25} \times \frac{8}{17} - \frac{7}{25} \times \frac{15}{17} = \frac{87}{425}$

Answer: $\frac{87}{425}$