Day 116 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "The sum of two irrational numbers is always irrational."

Use the numbers $5 + \sqrt{2}$ and $3 - \sqrt{2}$ to show this is false.

$(5 + \sqrt{2}) + (3 - \sqrt{2}) = 8$

Both numbers are irrational, but 8 is rational, so the statement is false.

Answer: sum = 8, which is rational

2 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_1^3 \frac{\ln x}{x}\,dx$,

giving your answer to 3 significant figures.

$h = \frac{3-1}{4} = 0.5$

x: 1, 1.5, 2, 2.5, 3

y: 0, 0.2703, 0.3466, 0.3665, 0.3662

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [0 + 0.3662 + 2 \times 0.9834] = 0.5833$

Answer: 0.583

3 Discrete random variable (AQA, OCR, MEI, Eduqas, CIE)

$P(X = x) = k(8 - x)$ for $x = 1, 2, ..., 6$.

(a) Find k.

(b) Find E(X) and Var(X).

(a) $k(7 + 6 + 5 + 4 + 3 + 2) = 1$, so $27k = 1$ and $k = \frac{1}{27}$

(b) $E(X) = 1 \times \frac{7}{27} + 2 \times \frac{2}{9} + 3 \times \frac{5}{27} + 4 \times \frac{4}{27} + 5 \times \frac{1}{9} + 6 \times \frac{2}{27} = \frac{77}{27}$

$E(X^2) = \frac{287}{27}$, so $Var(X) = \frac{287}{27} - (\frac{77}{27})^2 = \frac{1820}{729}$

Answer: (a) $k = \frac{1}{27}$ (b) $E(X) = \frac{77}{27}$, $Var(X) = \frac{1820}{729}$

4 Projectile: speed and direction at time t (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected horizontally at 294 m s^{-1} from a point high above the ground.

Take g = 9.8 m s^{-2}.

Find the speed and the direction of motion 16 s later, to 1 d.p. below the horizontal.

Vertical speed $= 9.8 \times 16 = 156.8$

Speed $= \sqrt{(294^2 + 156.8^2)} = 333.2$

$\tan \alpha = 156.8 \div 294$, $\alpha = 28.1°$

Answer: 333.2 m s^{-1} at 28.1° below horizontal

5 Geometric sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has second term −64 and fifth term 1.

Find (a) the common ratio, (b) the first term, (c) the sum to infinity.

$ar = -64$ and $ar^4 = 1$, so $r^3 = \frac{1}{-64} = -\frac{1}{64}$

$r = -\frac{1}{4}$, and $a = -64 \div (-\frac{1}{4}) = 256$

$S_\infty = \frac{a}{1-r} = \frac{256}{1-(-\frac{1}{4})} = \frac{1024}{5}$

Answer: (a) $r = -\frac{1}{4}$ (b) $a = 256$ (c) $\frac{1024}{5}$

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6 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from ground level with initial velocity components 14 m s^{-1} horizontally and 19.6 m s^{-1} vertically upwards.

Take g = 9.8 m s^{-2}.

Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 19.6^2 - 2 \times 9.8 \times H$, $H = 384.16 \div 19.6$

(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 19.6 m (b) 14 m s^{-1}

7 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write 2 cos θ + sin θ in the form R cos(θ − α), with R > 0 and 0 < α < 90°.

(b) Hence find its maximum value and the smallest θ ≥ 0 at which it occurs. Give α to 1 d.p.

$R = \sqrt{2^2 + 1^2} = \sqrt{5}$, $\tan \alpha = \frac{1}{2}$ so $\alpha = 26.6°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) $\sqrt{5}\cos(\theta - 26.6°)$ (b) max $\sqrt{5}$ at $\theta = 26.6°$

8 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = t + 3$, $y = 2t^2 + 3t - 1$.

The line y = x meets the curve at two points.

Find the coordinates of these points.

Substitute x and y into the line: $2t^2 + 3t - 1 = t + 3$

$2t^2 + 2t - 4 = 0$, so $2(t - 1)(t + 2) = 0$

$t = -2$ or $t = 1$, giving the points (1, 1) and (4, 4)

Answer: (1, 1) and (4, 4)

9 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(200, 0.3). Use a normal approximation to find P(X ≤ 56), to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(60.00, 42.00)$, so $\sigma = \sqrt{42.00} = 6.481$

Use a continuity correction.

$P(X \leq 56) \approx P(Y < 56.5) = P(Z < -0.5401) = 0.295$

Answer: 0.295

10 Quotient rule (All boards)

Given that $y = \frac{e^{2x}}{x + 2}$,

find $\frac{dy}{dx}$, and its value when x = 0.

Quotient rule with $u = e^{2x}$, $v = x + 2$

$u' = 2e^{2x}$, $v' = 1$

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(2x + 3)e^{2x}}{(x + 2)^2}$$

When x = 0: $\frac{dy}{dx} = \frac{3}{4}$

Answer: $\frac{dy}{dx} = \frac{(2x + 3)e^{2x}}{(x + 2)^2}$; value $\frac{3}{4}$