Day 115 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $2x^2 - 12x + 13$ in the form $a(x + p)^2 + q$.

(b) Write down the coordinates of the minimum point of $y = 2x^2 - 12x + 13$.

Take out the factor 2: $2x^2 - 12x + 13 = 2(x^2 - 6x) + 13$

$= 2[(x - 3)^2 - 9] + 13$

$= 2(x - 3)^2 - 5$

$(x - 3)^2 \geq 0$ with equality at $x = 3$, so the minimum is -5 at $x = 3$.

Answer: (a) $2(x - 3)^2 - 5$ (b) $(3, -5)$

2 Tangents and normals to exponential and log curves (All boards)

The curve C has equation $y = 3x^2 + e^{4x} - 3$.

Find the equation of the tangent to C at the point where $x = 0$, in the form $y = mx + c$.

$\frac{dy}{dx} = 6x + 4e^{4x}$

At $x = 0$: $y = -2$ and $\frac{dy}{dx} = 4$

$y + 2 = 4x$, so $y = 4x - 2$

Answer: $y = 4x - 2$

3 Velocity from variable acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line. At time t seconds its acceleration is $t - 4$ m s^{-2}.

When $t = 0$ the velocity is 7 m s^{-1}. Find the velocity when $t = 5$.

$v = \int (t - 4)\, dt = \frac{1}{2}t^2 - 4t + c$

$t = 0$, $v = 7$ so $c = 7$

$t = 5$: $v = \frac{-1}{2}$

Answer: -0.5 m s^{-1}

4 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int \frac{3}{5x + 3}\, dx$,

(b) the exact value of $\int_0^1 \frac{3}{5x + 3}\, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3 \ln(5x + 3)}{5} + c$

(b) $\left[\frac{3 \ln(5x + 3)}{5}\right]_0^1$

$= \frac{9 \ln 2}{5} - \frac{3 \ln 3}{5}$

Answer: (a) $\frac{3 \ln(5x + 3)}{5} + c$ (b) $\frac{9 \ln 2}{5} - \frac{3 \ln 3}{5}$

5 Two particles over a pulley (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles of mass 3 kg and 7 kg hang on a light string over a smooth fixed pulley, as shown, and are released from rest.

Take g = 9.8 m s^{-2}.

Find the acceleration and the tension in the string.

Heavier: $7g - T = 7a$

Lighter: $T - 3g = 3a$

Add: $4g = 10a$, so $a = 3.92$

$T = 3(9.8 + 3.92)$

Answer: $a = 3.92$ m s^{-2}; $T = 41.16$ N

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6 Triangle formed by two lines and an axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Line l_1 has equation $7x + 10y = 49$ and line l_2 has equation $7x + 2y = -7$.
The lines meet at P. Find the area of the triangle formed by l_1, l_2 and the x-axis.

Solve simultaneously: P = $(-3, 7)$
l_1 meets $y = 0$ at $x = 7$, and l_2 meets $y = 0$ at $x = -1$
Base = 8, height = 7
Area = $\frac{1}{2}$ × base × height = 28
Answer: 28

7 Differential equations in context (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cup of tea cools in a room at 15°C: $\frac{dT}{dt} = -k(T - 15)$. Initially T = 95 and after 5 minutes T = 60.
Find k to 3 s.f. and the time to cool to 25°C, to 3 s.f.

$\frac{1}{T - 15}\,dT = -k\,dt$, so $\ln(T - 15) = -kt + c$, i.e. $T = 15 + 80e^{-kt}$

$t = 5$: $60 = 15 + 80e^{-5k}$, so $e^{-5k} = \frac{9}{16}$

$k = \frac{1}{5}\ln\frac{80}{45} = 0.1151$

$25 = 15 + 80e^{-kt}$: $t = \frac{1}{k}\ln\frac{80}{10} = 18.07$

Answer: k = 0.115, t = 18.1 minutes

8 Tangent lines of a given gradient (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $3x + 4y = k$ is a tangent to the circle
$(x - 2)^2 + (y + 2)^2 = 100$.
Find the two possible values of k.

The radius to the point of contact is perpendicular to the line, so it is parallel to (3, 4).
A unit step in this direction is (3, 4)/5, so the points of contact are
$(2, -2) \pm 10 \times (3, 4)/5 = (2, -2) \pm (6, 8)$
T = (8, 6) or $(-4, -10)$, and $k = 3x + 4y$ gives:
k = 48 or k = −52
Answer: k = 48 or k = −52

9 Binomial critical region (All boards)

$X \sim B(25, p)$. H_0: $p = 0.3$ is to be tested against H_1: $p > 0.3$ at the 10% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(25, 0.3)$
$P(X \geq 11) = 0.0978 \leq 0.1$
$P(X \geq 10) = 0.1894 > 0.1$
The critical region is $X \geq 11$
Answer: X ≥ 11; actual significance level 9.78%

10 Normal approximation to the binomial (AQA, OCR, CIE)

$X \sim B(100, 0.5)$. Use a normal approximation to find $P(50 \leq X \leq 55)$, to 3 significant figures.

$Y \sim N(np, np(1 - p)) = N(50.00, 25.00)$, so $\sigma = \sqrt{25.00} = 5.000$
Use a continuity correction.
$P(50 \leq X \leq 55) \approx P(49.5 < Y < 55.5) = 0.404$
Answer: 0.404