Day 114 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

Points A and B have position vectors $\begin{pmatrix}-3\\-3\\5\end{pmatrix}$ and $\begin{pmatrix}-5\\-4\\3\end{pmatrix}$.

Find AB and the distance AB.

$\text{AB} = \text{OB} - \text{OA} = \begin{pmatrix}-2\\-1\\-2\end{pmatrix}$

$|\text{AB}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$

Answer: AB $= \begin{pmatrix}\mathbf{-2}\\\mathbf{-1}\\\mathbf{-2}\end{pmatrix}$**, distance 3**

2 Comparing box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plots show the times taken (minutes) to complete a puzzle for two groups.
(a) Find the interquartile range of each group.
(b) Compare the two distributions.

(a) Class A: 40 − 20 = 20. Class B: 55 − 25 = 30
(b) Medians: 35 (Class A) and 40 (Class B), so Class B has the higher median
IQRs: Class B has the larger IQR, so its scores are more spread out
Answer: (a) 20 and 30 (b) Class B has the higher median; Class B is more spread out (larger IQR), Class A more consistent

3 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 60 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 10 to t = 40.
The floors are 450 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (60 + 30) × V = 450
90 V = 900 so V = 10
Deceleration = 10 ÷ 20
Answer: V = 10; deceleration 0.5 m $\mathbf{s^{-2}}$

4 Solving exponential equations using logarithms (All boards)

Solve $e^{-3x} = 23$, giving your answer to 3 s.f.

Take natural logs: $-3x = \ln 23$
$x = \frac{\ln 23}{-3} = -1.045$
Answer: x = $\mathbf{-1.05}$

5 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

8 values of x are coded using $y = \frac{(x - 20)}{10}$.

$\Sigma y = -5$ and $\Sigma y^2 = 135$.
Find the mean and standard deviation of x (3 s.f.).

Mean of y = −5 ÷ 8 = −0.6250
Mean of x = 20 + 10 × −0.6250 = 13.8
Variance of y = $\frac{135}{8} - (-0.6250)^2 = 16.48$
SD of y = 4.060, so SD of x = 10 × 4.060 = 40.6
Answer: mean 13.8, standard deviation 40.6

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6 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of all the multiples of 6 that lie between 139 and 462.

First multiple = 6 × 24 = 144, last multiple = 6 × 76 = 456
This is an arithmetic series with d = 6 and 53 terms

$S = \frac{53}{2}(144 + 456) = 15900$

Answer: 15900

7 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $2\cos^2 x + 1 - 3\cos x = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

$(\cos x - 1)(2\cos x - 1) = 0$, so $\cos x = 1$ or $\cos x = \frac{1}{2}$

$\cos x = 1$: $x = 0.0°, 360.0°$

$\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

Answer: x = 0.0°, 60.0°, 300.0°, 360.0°

8 Work-energy: sliding to rest (CIE)

A block of mass 2 kg is projected along a rough horizontal surface with speed 2.8 m s^{-1} and comes to rest after sliding 4 m.
Take g = 9.8 m s^{-2}.
Use work and energy to find the coefficient of friction.

Work done against friction = loss of KE
$\mu \times 2 \times 9.8 \times 4 = ½ \times 2 \times 2.8^2$
$78.4\,\mu = 7.84$
Answer: 0.1

9 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + 2x)^{\frac{1}{2}}$ up to the x^2 term.
(b) Use x = 0.04 to estimate $\sqrt{1.08}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{2}$, a = 2:

$= 1 + x - \frac{1}{2}x^2$

(b) $1 + 2 \times 0.04 = 1.08$, so substitute x = 0.04:

$1 + 1 \times 0.04 - \frac{1}{2} \times 0.04^2 = 1.03920$

Answer: (a) $1 + x - \frac{1}{2}x^2 + \ldots$ (b) 1.03920

10 Transformations from a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). A and B are the turning points. Find the turning points of:
(a) y = f(x) − 2 (b) y = −f(x)

(a) y = f(x) − 2: A(1, −4) maps to (1, −6), B(5, 3) maps to (5, 1)
(b) y = −f(x): A(1, −4) maps to (1, 4), B(5, 3) maps to (5, −3)
Answer: (a) (1, −6), (5, 1) (b) (1, 4), (5, −3)