Day 113 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Particle on a table joined to a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 1 kg lies on a smooth horizontal table. It is attached by a light string over a smooth pulley at the edge of the table to a particle of mass 4 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released from rest. Find the acceleration and the tension in the string.

Hanging: $4g - T = 4a$

Table: $T = 1a$

$4g = 5a$, so $a = 7.84$

$T = 1 \times 7.84$

Answer: $a = 7.84$ m s^{-2}; $T = 7.84$ N

2 Basic trig equations (degrees) (All boards)

Solve $\cos 3x = -0.3$ for $0 \leq x \leq 360°$, giving answers to 1 d.p.

Let $\theta = 3x$, so $\cos\theta = -0.3$ with θ in range 0° to 1080°

$\theta = 107.46°, 252.54°, 467.46°, 612.54°, 827.46°, 972.54°$

$x = 35.8°, 84.2°, 155.8°, 204.2°, 275.8°, 324.2°$

Answer: $x = 35.8°, 84.2°, 155.8°, 204.2°, 275.8°, 324.2°$

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{4-\sqrt{2}}{\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are integers.

Multiply top and bottom by $\sqrt{2}$:

$\frac{(4-\sqrt{2})\sqrt{2}}{2} = \frac{4\sqrt{2}-2}{2}$

$= -1 + 2\sqrt{2}$

Answer: $-1 + 2\sqrt{2}$

4 Binomial one-tailed test (All boards)

A player says she scores with probability 0.5. In 30 shots she scores 13 times.

Test, at the 5% level, whether the true proportion is lower than claimed.

H_0: $p = 0.5$, H_1: $p < 0.5$. Let $X \sim B(30, 0.5)$ under H_0

$P(X \leq 13) = 0.2923$

$0.2923 > 0.05$, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

5 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 13e^x + 40 = 0$,

giving your answers as exact values.

Let $y = e^x$: $y^2 - 13y + 40 = 0$

$(y - 5)(y - 8) = 0$, so $e^x = 5$ or $e^x = 8$

$x = \ln 5$ or $x = \ln 8$

Answer: $x = \ln 5$, $x = \ln 8$

6 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations

$2x + y = -15$

$x^2 + y^2 = 225$

$y = -2x - 15$; substitute: $x^2 + (-2x - 15)^2 = 225$

$5x^2 + 60x = 0$, so $x^2 + 12x = 0$

$(x + 0)(x + 12) = 0$, so $x = 0$ or $x = -12$

$y = -15 - 2x$ gives $y = -15$ and $y = 9$

Answer: $x = -12$, $y = 9$ and $x = 0$, $y = -15$

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7 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A javelin is projected from level ground with horizontal velocity component 14 m s^{-1} and vertical velocity component 24.5 m s^{-1} upwards.

Take g = 9.8 m s^{-2}.

Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 24.5\,t - 4.9t^2$, so $t = 24.5 \div 4.9$

(b) Range = horizontal speed × time = 14 × 5

Answer: (a) 5 s (b) 70 m

8 Normal approximation to the binomial (AQA, OCR, CIE)

X ~ B(80, 0.6). Use a normal approximation to find P(X ≥ 49), to 3 significant figures.

Y ~ N(np, np(1 − p)) = N(48.00, 19.20), so $\sigma = \sqrt{19.20} = 4.382$

Use a continuity correction.

P(X ≥ 49) ≈ P(Y > 48.5) = P(Z > 0.1141) = 0.455

Answer: 0.455

9 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = \frac{1}{2}u_n + \frac{3}{2}$, $u_1 = 16$.

(a) Find u_2.

(b) Given that the sequence converges to a limit L, find L.

(a) $u_2 = \frac{1}{2} \times 16 + \frac{3}{2} = \frac{19}{2}$

(b) As $n \to \infty$, u_{n+1} and u_n both tend to L

$L = \frac{1}{2}L + \frac{3}{2}$

$L(1 - \frac{1}{2}) = \frac{3}{2}$, so L = 3

Answer: (a) $u_2 = \frac{19}{2}$ (b) L = 3

10 Stationary points of exponential and log curves (All boards)

Find the coordinates of the stationary point of the curve $y = 3x - \ln x$, and determine its nature.

$\frac{dy}{dx} = \frac{3x - 1}{x} = 0$

$x = \frac{1}{3}$

$\frac{d^2y}{dx^2} = \frac{1}{x^2}$

At $x = \frac{1}{3}$: $\frac{d^2y}{dx^2} = 9 > 0$, so a minimum

Answer: $(\frac{1}{3}, 1 + \ln 3)$ minimum