Day 112 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 7x + 46$ meets the circle $(x + 3)^2 + y^2 = 25$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = 7x + 46$: $(x + 3)^2 + (7x + 46)^2 = 25$

$x^2 + 13x + 42 = 0$

$(x + 6)(x + 7) = 0$, so $x = -7$ or $x = -6$

$y = 7x + 46$: A $= (-7, -3)$, B $= (-6, 4)$

$AB^2 = 1^2 + 7^2 = 50$

Answer: A$(-7, -3)$, B$(-6, 4)$, AB $= 5\sqrt{2}$

2 Independent and mutually exclusive events (All boards)

A and B are mutually exclusive events with $P(A) = 0.4$ and $P(A \cup B) = 1$.

(a) Find P(B).

(b) Find $P(A' \cap B')$.

(a) Mutually exclusive: $P(A \cup B) = P(A) + P(B)$

$P(B) = 1 - 0.4 = 0.6$

(b) $P(A' \cap B') = 1 - P(A \cup B) = 0$

Answer: (a) 0.6 (b) 0

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(4n + 2)^2 - (4n - 2)^2$ is a multiple of 32 for all integer values of n.

$(4n + 2)^2 = 16n^2 + 16n + 4$

$(4n - 2)^2 = 16n^2 - 16n + 4$

Difference $= 32n = 32 \times n$

n is an integer, so this is a multiple of 32 for every integer n.

Answer: difference = 32n, a multiple of 32

4 Impulse of a constant force: v = u + at with forces (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A puck of mass 8 kg is moving on a smooth horizontal surface at 8 m s^{-1} when a constant horizontal force of 4 N, in the direction of motion, acts for 4 s.

Find the speed of the puck at the end of this time and the distance it moves.

$F = ma$: $a = 4 \div 8 = 0.5$

$v = u + at = 8 + 0.5 \times 4$

$s = ut + \frac{1}{2}at^2$

Answer: 10 m s^{-1}; 36 m

5 Binomial one-tailed test (All boards)

A player says she scores with probability 0.4. In 15 shots she scores 4 times.

Test, at the 5% level, whether the true proportion is lower than claimed.

H_0: $p = 0.4$, H_1: $p < 0.4$. Let $X \sim B(15, 0.4)$ under H_0

$P(X \leq 4) = 0.2173$

$0.2173 > 0.05$, so do not reject H_0

There is insufficient evidence that the proportion is lower than claimed

Answer: do not reject H_0: insufficient evidence that the proportion is lower

6 Fixed-point iteration (All boards)

(a) Show that $x^3 - 3x - 6 = 0$ can be written as $x = \sqrt[3]{3x + 6}$.

(b) Use $x_{n+1} = \sqrt[3]{3x_n + 6}$ with $x_0 = 2$ to find x_1, x_2, x_3 to 4 d.p.

(a) $x^3 = 3x + 6$, so $x = \sqrt[3]{3x + 6}$

(b) $x_1 = 2.2894$

$x_2 = 2.3434$

$x_3 = 2.3531$

Answer: $x_1 = 2.2894$, $x_2 = 2.3434$, $x_3 = 2.3531$

7 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 3 kg lies on a rough horizontal table (coefficient of friction 0.25). It is joined by a light string over a smooth pulley to a particle of mass 2 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.25 \times 3g = 3a$

Hanging: $2g - T = 2a$

Add: $2g - 0.75g = 5a$

$a = 2.45$

$T = 2(9.8 - 2.45)$

Answer: $a = 2.45$ m s^{-2}; $T = 14.7$ N

8 Exponential quadratics and ln equations (All boards)

Solve $e^{2x} - 12e^x + 32 = 0$,

giving your answers as exact values.

Let $y = e^x$: $y^2 - 12y + 32 = 0$

$(y - 4)(y - 8) = 0$, so $e^x = 4$ or $e^x = 8$

$x = \ln 4$ or $x = \ln 8$

Answer: $x = \ln 4$, $x = \ln 8$

9 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 - x)^{-\frac{1}{2}}$ up to the x^2 term.

(b) Use $x = 0.01$ to estimate $\frac{1}{\sqrt{0.99}}$ to 5 decimal places.

(a) $(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = -\frac{1}{2}$, $a = -1$:

$= 1 + \frac{1}{2}x + \frac{3}{8}x^2$

(b) $1 - 1 \times 0.01 = 0.99$, so substitute $x = 0.01$:

$1 + \frac{1}{2} \times 0.01 + \frac{3}{8} \times 0.01^2 = 1.00504$

Answer: (a) $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \ldots$ (b) 1.00504

10 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{4}{5}$ and $\sin B = \frac{9}{41}$, where A and B are acute,

find the exact value of $\cos(A + B)$.

$\cos A = \frac{3}{5}$ and $\cos B = \frac{40}{41}$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \frac{3}{5} \times \frac{40}{41} - \frac{4}{5} \times \frac{9}{41} = \frac{84}{205}$

Answer: $\frac{84}{205}$