Day 111 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Distribution of the sample mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass, X g, of a loaf is normally distributed with mean 60 and s.d. 12.
A random sample of 36 is taken. Find the probability that the mean mass is less than 57 (3 s.f.).

$\overline{X}$ ~ N(60, $\frac{12^2}{36}$), standard deviation 2.000

$z = \frac{57 - 60}{2.000} = -1.500$

P = 0.0668

Answer: 0.0668

2 Product rule (All boards)

Given that $y = x^2e^{2x}$,
find $\frac{dy}{dx}$, and its value when x = 0.

Product rule with $u = x^2$, $v = e^{2x}$

$u' = 2x$, $v' = 2e^{2x}$

$\frac{dy}{dx} = u'v + uv' = 2x(x + 1)e^{2x}$

When x = 0: $\frac{dy}{dx} = 0$

Answer: $\frac{dy}{dx} = 2x(x + 1)e^{2x}$; value 0

3 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is the point (−6, −2) and B is the point (4, −1).
Find the perpendicular bisector of AB as ax + by + c = 0, with integer a, b, c.

Midpoint of AB = $(-1, -\frac{3}{2})$

Gradient of AB = $\frac{1}{10}$

Perpendicular gradient = −10

$y + \frac{3}{2} = -10(x + 1)$, which gives 20x + 2y + 23 = 0

Answer: 20x + 2y + 23 = 0

4 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 11 pairs of values, $\overline{x} = 5$, $\overline{y} = 18$, $S_{xx} = 20$ and $S_{xy} = 10$.
(a) Find the regression line of y on x in the form y = a + bx.
(b) Estimate y when x = 7.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{10}{20} = 0.5$

$a = \overline{y} - b\overline{x} = 18 - 0.5 \times 5 = 15.5$

y = 15.5 + 0.5x

(b) y = 15.5 + 0.5 × 7 = 19

Answer: y = 15.5 + 0.5x; y = 19

5 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 9 kg lies on a rough horizontal table (coefficient of friction 0.1). It is joined by a light string over a smooth pulley to a particle of mass 5 kg hanging freely.

Take g = 9.8 m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.1 \times 9\,g = 9\,a$

Hanging: $5\,g - T = 5\,a$

Add: $5\,g - 0.9\,g = 14\,a$

$a = 2.87$

$T = 5(9.8 - 2.87)$

Answer: a = 2.87 m s^{-2}; T = 34.65 N

6 Graph of the modulus of a quadratic (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), a quadratic.

(a) Find the coordinates of the local maximum point of y = |f(x)|.

(b) How many solutions has |f(x)| = 9?

(a) y = |f(x)| reflects the part of the curve below the x-axis in the x-axis:

(3, −9) becomes (3, 9)

(b) The maximum of the middle part is 9. Compare k = 9 with 9:

k = 9 gives 3 intersections with y = k.

Answer: (a) (3, 9) (b) 3

7 R cos(θ − α) form: solving (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $15\cos\theta + 8\sin\theta \equiv 17\cos(\theta - 28.1°)$,

solve $15\cos\theta + 8\sin\theta = -5$ for $0 \le \theta \le 360°$. (1 d.p.)

$17\cos(\theta - 28.1°) = -5$, so $\cos(\theta - 28.1°) = \frac{-5}{17}$

$\theta - 28.1° = \pm 107.1°$

$\theta = 28.1° + 107.1° = 135.2°$

$\theta = 28.1° - 107.1° = -79.0°$, so $\theta = 281.0°$

Answer: θ = 135.2°, 281.0°

8 Binomial expansion (positive integer power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In the binomial expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is −6 and the coefficient of x^2 is 15.

Find the values of n and k.

$nk = -6$, so $k = \frac{-6}{n}$

$\frac{n(n-1)}{2}k^2 = 15$, so $\frac{n(n-1)}{2} \times \frac{36}{n^2} = 15$

$\frac{(n-1) \times 36}{2n} = 15$, so $36(n - 1) = 30n$

$n = 6$, and $k = -1$

Answer: n = 6, k = −1

9 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\cot^2 x - \operatorname{cosec} x - 1 = 0$ for $0 \le x \le 360°$,

giving answers to 1 d.p.

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$: $2\operatorname{cosec}^2 x - \operatorname{cosec} x - 3 = 0$

$(2\operatorname{cosec} x - 3)(\operatorname{cosec} x + 1) = 0$, so $\operatorname{cosec} x = \frac{3}{2}$ or -1

$\operatorname{cosec} x = \frac{3}{2}$, so $\sin x = \frac{2}{3}$: $x = 41.8°, 138.2°$

$\operatorname{cosec} x = -1$, so $\sin x = -1$: $x = 270.0°$

Answer: x = 41.8°, 138.2°, 270.0°

10 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (2 kg) and B (12 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 3.5 m above the ground, then released.

Take $g = 9.8$ m s^{-2}.

Find the speed with which B hits the ground.

$a = (12 - 2)g \div (12 + 2) = 7$

$v^2 = 0 + 2 \times 7 \times 3.5 = 49$

$v = 7$

Answer: 7 m s^{-1}