Day 110 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moves in a straight line with constant acceleration. Its final speed is 41 m s^{-1}, its acceleration is 4 m s^{-2} and the motion lasts 10 s.
Find the initial speed.

$v = u + at$
$41 = u + 4 \times 10$
$u = 41 - 40$
Answer: 1 m s^{-1}

2 Independent and mutually exclusive events (All boards)

A and B are independent events with P(A) = 0.25 and P(A ∪ B) = 0.8125.
(a) Find P(B).
(b) Find P(A' ∩ B').

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$
$0.8125 = 0.25 + P(B)(1 - 0.25)$, so $P(B) = 0.5625 \div 0.75 = 0.75$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 0.1875$
Answer: (a) 0.75 (b) 0.1875

3 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $2\sin^2 x + 1 - \cos x = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $3 - 2\cos^2 x - \cos x = 0$, i.e. $2\cos^2 x + \cos x - 3 = 0$

$(2\cos x + 3)(\cos x - 1) = 0$, so $\cos x = -\frac{3}{2}$ or 1

$\cos x = -\frac{3}{2}$ has no solutions

$\cos x = 1$: $x = 0.0°, 360.0°$
Answer: x = 0.0°, 360.0°

4 Two independent binomials (All boards)

X ~ B(6, 0.3) and Y ~ B(12, 0.25) are independent.
Find P(X = 2 and Y ≤ 4), to 3 significant figures.

$P(X = 2) = 0.3241$
$P(Y \leq 4) = 0.8424$
Independent, so multiply: $0.3241 \times 0.8424 = 0.273$
Answer: 0.273

5 Intersection of a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = -x^2 - 3x + 1$ and the line $y = -4x - 1$ meet at the points A and B.
Find the coordinates of A and B.

$-x^2 - 3x + 1 = -4x - 1$
$-x^2 + x + 2 = 0$, so $-(x + 1)(x - 2) = 0$
$x = -1$ or $x = 2$
$y = -4x - 1$: $y = 3$ and $y = -9$
Answer: A(−1, 3), B(2, −9)

6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations

to find the value of $\lim_{\theta \to 0} \frac{\tan\theta \sin 3\theta}{1 - \cos 2\theta}$.

$\tan\theta \sin 3\theta \approx \theta \times 3\theta = 3\theta^2$

$1 - \cos 2\theta \approx \frac{4\theta^2}{2}$

Ratio $= \frac{3\theta^2}{\frac{4\theta^2}{2}} = \frac{3}{2}$

Answer: $\mathbf{\frac{3}{2}}$

7 Displacement from a v-t formula; average velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves along a straight line so that its displacement from O after t s is $s = 2t^3 + 4t^2$ metres.
Find (a) the average velocity over the first 6 seconds, (b) the velocity at t = 6.

(a) $s(6) = 576$, so average velocity $= 576 \div 6$

(b) $v = ds/dt = 6t^2 + 8t$, $v(6) = 264$

Answer: (a) 96 m s^{-1} (b) 264 m s^{-1}

8 Collinear points and parallel vectors (All boards)

Points A, B and C have position vectors $\begin{pmatrix}3\\1\\0\end{pmatrix}$, $\begin{pmatrix}2\\3\\0\end{pmatrix}$ and $\begin{pmatrix}4\\q\\0\end{pmatrix}$ respectively.

Given that A, B, C are collinear, find q.

$\text{AB} = \begin{pmatrix}-1\\2\\0\end{pmatrix}$, $\text{AC} = \begin{pmatrix}1\\q-1\\0\end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 1 gives $1 = \lambda \times -1$, so $\lambda = -1$

$q - 1 = -1 \times 2$, so $q = -1$

Answer: $q = -1$

9 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -2x - 25$ meets the circle $(x + 3)^2 + (y - 1)^2 = 100$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -2x - 25$: $(x + 3)^2 + (-2x - 26)^2 = 100$

$x^2 + 22x + 117 = 0$

$(x + 9)(x + 13) = 0$, so $x = -13$ or $x = -9$

$y = -2x - 25$: A $= (-13, 1)$, B $= (-9, -7)$

$\text{AB}^2 = 4^2 + (-8)^2 = 80$

Answer: A$(-13, 1)$, B$(-9, -7)$, AB $= 4\sqrt{5}$

10 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 3x^3 + px^2 + qx - 9$.

When f(x) is divided by $(x - 2)$ the remainder is -11, and when divided by $(x - 3)$ the remainder is 21. Find p and q.

$f(2) = -11$: $24 + 4p + 2q - 9 = -11$, so $4p + 2q = -26$

$f(3) = 21$: $81 + 9p + 3q - 9 = 21$, so $9p + 3q = -51$

Solving simultaneously: $p = -4$, $q = -5$

Answer: $p = -4$, $q = -5$