Day 11 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -10 \\ 2 \\ -11 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{10^2 + 2^2 + 11^2} = \sqrt{225} = 15$

Unit vector $= \frac{1}{15}\mathbf{a}$

Answer: $|\mathbf{a}| = 15$, unit vector $\frac{1}{15}\begin{pmatrix} -10 \\ 2 \\ -11 \end{pmatrix}$

2 Constant acceleration (suvat) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sprinter moves in a straight line with constant acceleration. Its initial speed is 16 m s^{-1}, its final speed is 46 m s^{-1} and the motion lasts 10 s.

Find the distance travelled.

$s = \frac{1}{2}(u + v)t$

$s = \frac{1}{2} \times (16 + 46) \times 10$

Answer: 310 m

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Statement: "The sum of two irrational numbers is always irrational."

Use the numbers $6 + \sqrt{5}$ and $4 - \sqrt{5}$ to show this is false.

$(6 + \sqrt{5}) + (4 - \sqrt{5}) = 10$

Both numbers are irrational, but 10 is rational, so the statement is false.

Answer: sum = 10, which is rational

4 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.

(a) Describe the correlation.

(b) Why should $y = 2 + x$ not be used for $x = 15$?

(a) The points lie loosely around a straight line rising: weak positive correlation

(b) The data cover $x = 1$ to 10; $x = 15$ is outside this range

so using the line would be extrapolation and may be unreliable

Answer: (a) weak positive correlation (b) extrapolation outside the range of the data

5 Volume of revolution about the x-axis (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The region R is bounded by the curve $y = \frac{1}{x}$, the x-axis

and the lines $x = 1$ and $x = 2$.

R is rotated through 360° about the x-axis. Find the exact volume generated.

$V = \pi \int_1^2 y^2 \, dx = \pi \int_1^2 \frac{1}{x^2} \, dx$

$= \pi\left[-\frac{1}{x}\right]_1^2$

$= \frac{\pi}{2}$

Answer: $\frac{\pi}{2}$

6 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + 2x^2 + kx - 18$.
Given that $(x + 3)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(-3) = 0$: $-27 + 18 - 3k - 18 = 0$
$k = -9$
$f(x) = (x + 3)(x^2 - x - 6)$
$= (x - 3)(x + 2)(x + 3)$
Answer: (a) $k = -9$ (b) $f(x) = (x - 3)(x + 2)(x + 3)$

7 Two independent binomials (All boards)

$X \sim B(7, 0.4)$ and $Y \sim B(12, 0.1)$ are independent.
Find $P(X = 3 \text{ and } Y \leq 1)$, to 3 significant figures.

$P(X = 3) = 0.2903$
$P(Y \leq 1) = 0.6590$
Independent, so multiply: $0.2903 \times 0.6590 = 0.191$
Answer: 0.191

8 Geometric series and logarithms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 8 and common ratio 4.
Find the smallest value of n for which the sum of the first n terms exceeds 500.

$S_n = \frac{8(4^n - 1)}{3} > 500$
$4^n > 188.5$
$n > \frac{\lg 188.5}{\lg 4} = 3.779$
So the smallest integer n is 4
Answer: $n = 4$

9 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int (2x + 6)^4 \, dx$,
(b) the exact value of $\int_1^2 (2x + 6)^4 \, dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{(2x + 6)^5}{10} + c$
(b) $\left[\frac{(2x + 6)^5}{10}\right]_1^2$
$= \frac{33616}{5}$
Answer: (a) $\frac{(2x + 6)^5}{10} + c$ (b) $\frac{33616}{5}$

10 Rod suspended by two vertical strings (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform rod AB of length 3 m and mass 10 kg hangs horizontally from two vertical strings attached 0.5 m from A and 0.5 m from B.
A particle of mass 2 kg is attached 1 m from A. Take $g = 9.8$ m s^{-2}. Find the tension in each string.

Moments about the string near A: $T_2(2) = 10\,g(1) + 2\,g(0.5)$
$T_2 = 53.9$
$T_1 + T_2 = 12\,g$ so $T_1 = 117.6 - 53.9$
Answer: Near A: 63.7 N; near B: 53.9 N