Day 109 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 + px^2 + qx - 3$.
When f(x) is divided by (x + 2) the remainder is −35, and when divided by (x + 3) the remainder is −78. Find p and q.

$f(-2) = -35$: $-8 + 4p - 2q - 3 = -35$, so $4p - 2q = -24$
$f(-3) = -78$: $-27 + 9p - 3q - 3 = -78$, so $9p - 3q = -48$
Solving simultaneously: $p = -4$, $q = 4$
Answer: $p = -4$, $q = 4$

2 Fixed-point iteration (All boards)

(a) Show that $e^x - x - 3 = 0$ can be written as $x = \ln(x + 3)$.
(b) Use $x_{n+1} = \ln(x_n + 3)$ with $x_0 = 1$ to find x_1, x_2, x_3 to 4 d.p.

(a) $e^x = x + 3$, so $x = \ln(x + 3)$
(b) $x_1 = 1.3863$
$x_2 = 1.4785$
$x_3 = 1.4993$
Answer: $x_1 = 1.3863$, $x_2 = 1.4785$, $x_3 = 1.4993$

3 Binomial expected frequency (All boards)

A biased coin with P(head) = $\frac{1}{6}$ is tossed 8 times; X is the number of heads.

This is done 2000 times. Estimate the number of times that X = 2.

$X \sim B(8, \frac{1}{6})$

$P(X = 2) = {}^8C_2 \times (\frac{1}{6})^2 \times (\frac{5}{6})^6 = 0.2605$

Expected = $2000 \times 0.2605 = 521.0$
Answer: about 521 times

4 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the term independent of x in the expansion of

$(x^2 + \frac{1}{x})^6$.

The general term is $C(6, k)(x^2)^{6-k}(1/x)^k$, with power of x equal to $12 - 3k$
Independent of x: $12 - 3k = 0$, so $k = 4$
Term = $15 \times 1^4 = 15$
Answer: 15

5 Projection from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 9.8 m s^{-1} from a point 39.2 m above horizontal ground.
Take g = 9.8 m s^{-2}.
Find the time before it hits the ground.

Taking up as positive: $-39.2 = 9.8\,t - 4.9t^2$
$4.9t^2 - 9.8\,t - 39.2 = 0$
$t = (9.8 \pm \sqrt{864.36}) \div 9.8 = (9.8 \pm 29.4) \div 9.8$
Positive root only.
Answer: 4 s

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6 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $xe^y - 3y = 2$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient at (2, 0).

Differentiate with respect to x: $x\frac{dy}{dx}e^y + e^y - 3\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{e^y}{xe^y - 3}$

At (2, 0): gradient = 1

Answer: $\frac{dy}{dx} = -\frac{e^y}{xe^y - 3}$; gradient 1

7 Lift: weighing scales reading (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A woman of mass 40 kg stands on weighing scales in a lift. The lift decelerates at 1 m s^{-2} while moving upwards.

Take g = 9.8 m s^{-2}.

Find the force exerted on the woman by the scales.

The acceleration is downwards, magnitude 1

Taking up as positive: $R - 40 \times 9.8 = -40 \times 1$

$R = 40(9.8 - 1)$

Answer: 352 N

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Given that $\sqrt{5}$ is irrational, prove by contradiction that $1 + 5\sqrt{5}$ is irrational.

Assume $1 + 5\sqrt{5} = \frac{r}{s}$, where r and s are integers, $s \neq 0$.

Then $5\sqrt{5} = \frac{r}{s} - 1 = \frac{r - 1s}{s}$, so $\sqrt{5} = \frac{r - 1s}{5s}$.

The right-hand side is a ratio of integers, so it is rational.

This contradicts $\sqrt{5}$ being irrational, so $1 + 5\sqrt{5}$ is irrational.

Answer: $1 + 5\sqrt{5}$ is irrational

9 Rates of change (connected rates) (All boards)

The volume of a sphere increases at a constant rate of 12 cm^3s^{-1}.

Find the rate at which the radius is increasing when r = 6 cm, to 3 s.f.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV} = \frac{12}{4\pi r^2}$

When r = 6: $\frac{dr}{dt} = \frac{1}{12\pi} = 0.02653$

Answer: 0.0265 cm/s

10 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

9 values of x are coded using $y = \frac{(x - 100)}{2}$.

$\Sigma y = 8$ and $\Sigma y^2 = 224$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = 8 \div 9 = 0.8889$

Mean of $x = 100 + 2 \times 0.8889 = 102$

Variance of $y = \frac{224}{9} - (0.8889)^2 = 24.10$

SD of y = 4.909, so SD of $x = 2 \times 4.909 = 9.82$

Answer: mean 102, standard deviation 9.82