Day 108 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Quadratic equations in sin x, cos x, tan x (All boards)

Solve $\tan^2 x - 2\tan x - 3 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

$(\tan x + 1)(\tan x - 3) = 0$, so $\tan x = -1$ or $\tan x = 3$
$\tan x = -1$: $x = 135.0°, 315.0°$
$\tan x = 3$: $x = 71.6°, 251.6°$
Answer: x = 71.6°, 135.0°, 251.6°, 315.0°

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(5x^2)^3 \times 6x^4}{2x^2}$.

$(5x^2)^3 = 125x^6$
Numerator $= 125 \times 6x^{10} = 750x^{10}$
Divide: $\frac{750}{2} = 375$ and $x^{10} \div x^2 = x^8$
Answer: $375x^8$

3 Integrating composite functions (reverse chain rule) (All boards)

Find (a) $\int 3\sec^2 5x\,dx$,
(b) the exact value of $\int_0^{\frac{\pi}{20}} 3\sec^2 5x\,dx$.

(a) Reverse the chain rule and divide by the coefficient of x: $\frac{3\tan 5x}{5} + c$
(b) $\left[\frac{3\tan 5x}{5}\right]_0^{\frac{\pi}{20}}$
$= \frac{3}{5}$
Answer: (a) $\frac{3\tan 5x}{5} + c$ (b) $\frac{3}{5}$

4 Projectile at an angle: time and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected from level ground with horizontal velocity component 11 m s^{-1} and vertical velocity component 24.5 m s^{-1} upwards.
Take g = 9.8 m s^{-2}.
Find (a) the time of flight, (b) the horizontal range.

(a) Vertical: $0 = 24.5t - 4.9t^2$, so $t = 24.5 \div 4.9$
(b) Range = horizontal speed × time $= 11 \times 5$
Answer: (a) 5 s (b) 55 m

5 Regression line (AQA, Edexcel, OCR, MEI, Eduqas)

For 14 pairs of values, $\bar{x} = 6$, $\bar{y} = 25$, $S_{xx} = 50$ and $S_{xy} = -50$.
(a) Find the regression line of y on x in the form $y = a + bx$.
(b) Estimate y when x = 4.

(a) $b = \frac{S_{xy}}{S_{xx}} = \frac{-50}{50} = -1$
$a = \bar{y} - b\bar{x} = 25 - -1 \times 6 = 31$
$y = 31 - x$
(b) $y = 31 + (-1) \times 4 = 27$
Answer: y = 31 − x; y = 27

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6 Finding the equation of a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of the curve $y = f(x)$, where f is a cubic. The curve meets the axes at the labelled points. Find $f(x)$ in factorised form.

x-intercepts at $x = -4, -1$ and 3, so $y = k(x + 4)(x + 1)(x - 3)$
The curve passes through $(0, -24)$: $k \times 4 \times 1 \times (-3) = -24$
So $k = 2$
Answer: $f(x) = 2(x + 4)(x + 1)(x - 3)$

7 Exponential and power models (log-linear) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Data are modelled by $y = ax^n$. A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with intercept 0.477 and gradient 0.5.
Find a (2 s.f.) and n, then estimate y when $x = 4$ (3 s.f.).

$\log_{10} y = \log_{10} a + n \log_{10} x$
n = gradient = 0.5; $\log_{10} a = 0.477$, so $a = 10^{0.477} = 3.0$
$y = 2.999 \times 4^{0.5} = 6.00$
Answer: $a = 3.0$, $n = 0.5$, $y = 6.00$

8 Equations using $\sin^2 x + \cos^2 x = 1$ (All boards)

Solve $2\sin^2 x + \cos x - 1 = 0$ for $0 \le x \le 360°$, giving answers to 1 d.p. where necessary.

Use $\sin^2 x = 1 - \cos^2 x$: $\cos x + 1 - 2\cos^2 x = 0$, i.e. $2\cos^2 x - \cos x - 1 = 0$
$(\cos x - 1)(2\cos x + 1) = 0$, so $\cos x = 1$ or $-\frac{1}{2}$
$\cos x = 1$: $x = 0.0°, 360.0°$
$\cos x = -\frac{1}{2}$: $x = 120.0°, 240.0°$
Answer: $x = 0.0°, 120.0°, 240.0°, 360.0°$

9 Pulley system: speed on reaching the ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (3 kg) and B (12 kg) are joined by a light string over a smooth pulley. Both are held with the string taut, B is 6 m above the ground, then released.
Take $g = 9.8$ m s^{-2}.
Find the speed with which B hits the ground.

$a = (12 - 3)g \div (12 + 3) = 5.88$
$v^2 = 0 + 2 \times 5.88 \times 6 = 70.56$
$v = 8.4$
Answer: 8.4 m s^{-1}

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers m and n such that $28m + 21n = 58$.

Assume there are integers m and n with $28m + 21n = 58$.
$28m + 21n = 7(4m + 3n)$.
So the left-hand side is a multiple of 7.
But $58 = 7 \times 8 + 2$ is not a multiple of 7.
This is a contradiction, so no such integers exist.
Answer: no integer solutions (LHS is a multiple of 7, 58 is not)