Day 107 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Arc length and sector area (radians) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sector OAB of a circle with centre O, radius 8 cm and angle 0.7 radians.
Find (a) the arc length AB, (b) the area of the sector.

Arc length = $r\theta = 8 \times 0.7 = 5.6$ cm

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times 0.7 = 22.4$ cm^2

Answer: (a) 5.6 cm (b) 22.4 cm^2

2 Equation of a line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A and B have coordinates $(-4, -1)$ and $(1, -2)$.
(a) Find the midpoint of AB.
(b) Find the line AB as $ax + by + c = 0$, with integer a, b, c.

(a) Midpoint = $(\frac{-4+1}{2}, \frac{-1+(-2)}{2}) = (-\frac{3}{2}, -\frac{3}{2})$

(b) Gradient = $\frac{-1}{5} = -\frac{1}{5}$

$y + 1 = -\frac{1}{5}(x + 4)$

Multiply out and collect terms: $x + 5y + 9 = 0$

Answer: (a) $(-\frac{3}{2}, -\frac{3}{2})$ (b) $x + 5y + 9 = 0$

3 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.
Find (a) the acceleration in the first 10 s, (b) the distance between the stations.

(a) a = gradient = 12 ÷ 10
(b) Distance = area of trapezium = ½ × (35 + 15) × 12

Answer: (a) 1.2 m s^{-2} (b) 300 m

4 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\frac{(4x^3)^3 \times 6x^3}{8x^6}$.

$(4x^3)^3 = 64x^9$

Numerator = $64 \times 6x^{12} = 384x^{12}$

Divide: $\frac{384}{8} = 48$ and $x^{12} \div x^6 = x^6$

Answer: $48x^6$

5 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. Give α to 1 d.p.

$R\cos\alpha = 3$, $R\sin\alpha = 4$

$R = \sqrt{3^2 + 4^2} = 5$

$\tan\alpha = \frac{4}{3}$, so $\alpha = 53.1°$

Answer: $5\cos(\theta - 53.1°)$

6 Speed-time graph: find the maximum speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift travels between floors in 75 s. The speed-time graph is a trapezium, with the maximum speed V m s^{-1} held from t = 5 to t = 65.
The floors are 405 m apart. Find V, and the deceleration in the final stage.

Area = ½ × (75 + 60) × V = 405
135 V = 810 so V = 6
Deceleration = 6 ÷ 10

Answer: V = 6; deceleration 0.6 m s^{-2}

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7 Binomial critical region (All boards)

$X \sim B(20, p)$. H_0: $p = 0.25$ is to be tested against H_1: $p < 0.25$ at the 5% significance level.
Find the critical region and the actual significance level.

Under H_0, $X \sim B(20, 0.25)$
$P(X \leq 1) = 0.0243 \leq 0.05$
$P(X \leq 2) = 0.0913 > 0.05$
The critical region is $X \leq 1$
Answer: $X \leq 1$; actual significance level 2.43%

8 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t + 3$, $y = -t^2 - 5t - 3$.
The line $y = 2x - 9$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 - 5t - 3 = -2t - 3$
$-t^2 - 3t = 0$, so $-t(t + 3) = 0$
$t = -3$ or $t = 0$, giving the points (6, 3) and (3, −3)
Answer: (6, 3) and (3, −3)

9 Compound angle formulae (exact values) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Given that $\sin A = \frac{24}{25}$ and $\sin B = \frac{9}{41}$, where A and B are acute,
find the exact value of $\sin(A + B)$.

$\cos A = \frac{7}{25}$ and $\cos B = \frac{40}{41}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$= \frac{24}{25} \times \frac{40}{41} + \frac{7}{25} \times \frac{9}{41} = \frac{1023}{1025}$
Answer: $\frac{1023}{1025}$

10 Binomial: smallest n (All boards)

Given that each lottery ticket wins a prize independently with probability 0.4, find the least number of tickets needed for the probability of at least one success to exceed 0.95.

$P(\text{at least one}) = 1 - 0.6^n > 0.95$
$0.6^n < 0.05$
$n > \frac{\log 0.05}{\log 0.6} = 5.864$
Answer: n = 6