Day 106 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A sports club has 220 junior members, 260 senior members and 100 veteran members.
A stratified sample of size 29 is taken. Find the number selected from each group.

Total = 220 + 260 + 100 = 580

Sampling fraction = 29/580 = $\frac{1}{20}$

junior: 220 ÷ 20 = 11
senior: 260 ÷ 20 = 13
veteran: 100 ÷ 20 = 5

Answer: junior 11, senior 13, veteran 5

2 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^3 + 7n + 12$ is even for all integers n.

$n^3 + 7n = n(n^2 + 7)$

If n is even, $n(n^2 + 7)$ is even.

If n is odd, n^2 is odd and $n^2 + 7$ is even, so $n(n^2 + 7)$ is even.

So $n^3 + 7n$ is always even, and adding 12 gives an even number.

Answer: expression is even

3 Binomial expected frequency (All boards)

A biased coin with P(head) = 0.5 is tossed 6 times; X is the number of heads.
This is done 200 times. Estimate the number of times that X = 4.

X ~ B(6, 0.5)

$P(X = 4) = {}^6C_4 \times (0.5)^4 \times (0.5)^2 = 0.2344$

Expected = 200 × 0.2344 = 46.88

Answer: about 47 times

4 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = \frac{6y}{x}$ for x > 0,

given that y = 4 when x = 1.

$\frac{1}{y}\,dy = \frac{6}{x}\,dx$, so $\ln y = 6 \ln x + c$

x = 1, y = 4: c = ln 4

$y = 4x^6$

Answer: $y = 4x^6$

5 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $2x^2 - 16x + 34 > 0$ for all real values of x.

Complete the square: $2x^2 - 16x + 34 = 2(x - 4)^2 + 2$

$(x - 4)^2 \geq 0$ for all real x, so $2(x - 4)^2 \geq 0$.

Adding 2: the expression is ≥ 2, so > 0 for all real x.

Answer: $2x^2 - 16x + 34 > 0$ for all x

6 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 8t - 4t^2$ m s^{-1}, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

a = dv/dt = 8 − 8t

Maximum speed when a = 0: t = 8 ÷ 8 = 1

$v = 8 \times 1 - 4 \times 1^2$

Answer: 4 m s^{-1} at t = 1 s

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7 Trapezium rule (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^2 \frac{1}{1+x^2}\,dx$,

giving your answer to 3 significant figures.

$h = \frac{2-0}{4} = 0.5$

x: 0, 0.5, 1, 1.5, 2

y: 1.000, 0.8000, 0.5000, 0.3077, 0.2000

Area $\approx \frac{h}{2}[y_0 + y_4 + 2(y_1 + \ldots + y_3)]$

$= \frac{0.5}{2} \times [1.000 + 0.2000 + 2 \times 1.608] = 1.104$

Answer: 1.10

8 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle of mass 1 kg is suspended in equilibrium by two light strings attached to a horizontal ceiling. The strings make angles α and β with the ceiling, where $\tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{3}{4}$.

Take $g = 9.8$ m s^{-2}.

Find the tension in each string.

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, $T_1 \times \frac{3}{5} = T_2 \times \frac{4}{5}$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 9.8$

From the first: $T_1 = \frac{4}{3} T_2$; into the second: $T_2 \times \frac{5}{3} = 9.8$

Answer: T_1 = 7.84 N (at α); T_2 = 5.88 N (at β)

9 Locating roots: change of sign (All boards)

The function $f(x) = e^x - 5x$.

Show that $f(x) = 0$ has a root between $x = 2.5$ and $x = 3$.

$f(2.5) = -0.318$

$f(3) = 5.09$

f is continuous and changes sign, so there is a root in the interval

Answer: f(2.5) < 0 and f(3) > 0: sign change

10 Coefficients in a binomial expansion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^3 in the expansion of $(1 + 3x)^8$.

Term in $x^3 = 56 \times 1^5 \times (3x)^3$

$= 56 \times 1 \times 27\,x^3$

Answer: 1512