Day 105 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas)

Simplify $\frac{4}{\sqrt{2}} + \sqrt{32}$, giving your answer in the form $k\sqrt{2}$.

$\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\sqrt{32} = 4\sqrt{2}$

$2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$

Answer: $6\sqrt{2}$

2 Driving force and resistance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car of mass 1000 kg moves along a straight horizontal road. The engine produces a driving force of 3500 N and the resistance to motion is 900 N.

Find the acceleration of the car.

Resultant force = $3500 - 900 = 2600$ N

F = ma: $2600 = 1000\,a$

Answer: 2.6 m s^{-2}

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x - 4$ and $g(x) = x^2 - 3$ for $x \in \mathbb{R}$.

(a) Find fg(x).

(b) Solve gf(x) = 141.

$fg(x) = f(x^2 - 3) = 2(x^2 - 3) - 4 = 2x^2 - 10$

$gf(x) = (2x - 4)^2 - 3 = 141$

$(2x - 4)^2 = 144$, so $2x - 4 = \pm 12$

$x = -4$ or $x = 8$

Answer: (a) $fg(x) = 2x^2 - 10$ (b) $x = -4$ or $x = 8$

4 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

9 values of x are coded using $y = \frac{(x - 100)}{2}$.

$\Sigma y = -2$ and $\Sigma y^2 = 158$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = -2 \div 9 = -0.2222$

Mean of $x = 100 + 2 \times -0.2222 = 99.6$

Variance of $y = \frac{158}{9} - (-0.2222)^2 = 17.51$

SD of y = 4.184, so SD of $x = 2 \times 4.184 = 8.37$

Answer: mean 99.6, standard deviation 8.37

5 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas)

A sequence is defined by $u_{n+1} = -2u_n + 5$, $u_1 = -2$.

(a) Find u_2 and u_3.

(b) Find $\sum_{r=1}^{4} u_r$.

(a) $u_2 = -2 \times (-2) + 5 = 9$

$u_3 = -2 \times 9 + 5 = -13$

(b) $u_4 = -2 \times (-13) + 5 = 31$

Sum $= -2 + 9 + (-13) + 31 = 25$

Answer: (a) $u_2 = 9$, $u_3 = -13$ (b) 25

6 R cos(θ − α) form: express and maximum (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write cos θ + 2 sin θ in the form R cos(θ − α), with R > 0 and 0 < α < 90°.
(b) Hence find its maximum value and the smallest θ ≥ 0 at which it occurs. Give α to 1 d.p.

$R = \sqrt{1^2 + 2^2} = \sqrt{5}$, $\tan \alpha = \frac{2}{1}$ so $\alpha = 63.4°$

Maximum is R when $\cos(\theta - \alpha) = 1$, i.e. $\theta = \alpha$

Answer: (a) $\sqrt{5}\cos(\theta - 63.4°)$ (b) max $\sqrt{5}$ at $\theta = 63.4°$

7 Projectile: greatest height and speed at the top (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected from ground level with initial velocity components 15 m s^{-1} horizontally and 39.2 m s^{-1} vertically upwards.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height, (b) the speed at the greatest height.

(a) $0 = 39.2^2 - 2 \times 9.8 \times H$, $H = 1536.64 \div 19.6$
(b) At the top the vertical velocity is zero, so the speed is the horizontal component.

Answer: (a) 78.4 m (b) 15 m s^{-1}

8 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{2}} (\cos^2 x + 1 - \sin^2 x)\, dx$.

Use $\cos^2 x - \sin^2 x = \cos 2x$

$\int (\cos^2 x + 1 - \sin^2 x)\, dx = x + \frac{\sin 2x}{2}$

$= \left[x + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}$

Answer: $\frac{\pi}{2}$

9 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 38) = 0.0668 and P(X > 58) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$38 = \mu + (-1.500)\sigma$ and $58 = \mu + 0.9998\sigma$
Subtract: $20 = 2.500\,\sigma$, so $\sigma = 8.00$
$\mu = 38 - (-1.500)(8.000) = 50.0$

Answer: $\mu = 50.0$, $\sigma = 8.00$

10 Solving exponential equations using logarithms (All boards)

Solve $5^x = 35$, giving your answer to 3 s.f.

Take logs: $x \ln 5 = \ln 35$

$x = \frac{\ln 35}{\ln 5} = 2.209$

Answer: x = 2.21