Day 104 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Proof using parity (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that if n is odd then $n^2 + 3n + 7$ is odd.

Let n = 2k + 1, where k is an integer.

$n^2 = 4k^2 + 4k + 1$

$3n = 6k + 3$

Total = $4k^2 + 10k + 11$

4k + 10 is even and 11 is odd, so the total is odd.

Answer: the expression is odd

2 Separable differential equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{dy}{dx} = 6xy$, given that y = 3 when x = 0.

Give y in terms of x.

$\frac{1}{y}\,dy = 6x\,dx$, so $\ln y = \frac{6x^2}{2} + c$

x = 0, y = 3: c = ln 3

$y = 3\,e^{3x^2}$

Answer: $y = 3e^{3x^2}$

3 Newton's second law: lift and tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg is raised and lowered by a vertical cable. The lift accelerates downwards at 0.25 m s^{-2}.

Take g = 9.8 m s^{-2}.

Find the tension in the cable.

Resolve vertically (up positive): $T - 250 \times 9.8 = -250 \times 0.25$

$T = 250(9.8 - 0.25)$

Answer: 2387.5 N

4 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the quotient and the remainder when

$4x^3 + 4x^2 - 5x - 3$ is divided by $(2x - 1)$.

$2x^2 \times (2x - 1) = 4x^3 - 2x^2$, leaving $6x^2 - 5x - 3$

$3x \times (2x - 1) = 6x^2 - 3x$, leaving $-2x - 3$

$-1 \times (2x - 1) = -2x + 1$, leaving -4

Quotient $2x^2 + 3x - 1$, remainder -4

Answer: quotient $2x^2 + 3x - 1$, remainder -4

5 Beam on two supports: reactions (AQA, Edexcel, OCR, MEI, Eduqas)

A uniform beam AB of length 5 m and mass 10 kg rests horizontally on supports at C and D, where AC = 2 m and BD = 0.5 m.

A particle of mass 15 kg is placed on the beam 2 m from B. Take g = 9.8 m s^{-2}.

Find the reactions at C and D.

Moments about C: $R_D \times 2.5 = 10\,g \times 0.5 + 15\,g \times 1$

$R_D = 196 \div 2.5$

Resolve vertically: $R_C + R_D = 25\,g$

Answer: R_C = 166.6 N; R_D = 78.4 N

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6 Equations using $\sin^2x + \cos^2x = 1$ (All boards)

Solve $6\sin^2x + 5\cos x - 7 = 0$ for $0 \le x \le 360°$,
giving answers to 1 d.p. where necessary.

Use $\sin^2x = 1 - \cos^2x$: $5\cos x - 6\cos^2x - 1 = 0$, i.e. $6\cos^2x + 1 - 5\cos x = 0$

$(3\cos x - 1)(2\cos x - 1) = 0$, so $\cos x = \frac{1}{3}$ or $\frac{1}{2}$

$\cos x = \frac{1}{3}$: $x = 70.5°, 289.5°$

$\cos x = \frac{1}{2}$: $x = 60.0°, 300.0°$

Answer: $x = 60.0°, 70.5°, 289.5°, 300.0°$

7 Perpendicular lines and shortest distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = -x$. The point P is $(-4, 2)$.
(a) Find the line through P perpendicular to l, as $ax + by + c = 0$ with integer a, b, c.
(b) Find the foot F and the exact distance PF.

(a) Gradient of l $= -1$, so the perpendicular has gradient 1
$y - 2 = (x + 4)$, giving $x - y + 6 = 0$
(b) Solve $y = -x$ with $x - y + 6 = 0$: $F = (-3, 3)$
$PF^2 = (-1)^2 + (-1)^2 = 2$

Answer: (a) $x - y + 6 = 0$ (b) $F(-3, 3)$, $PF = \sqrt{2}$

8 Histogram bar heights (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A class of width 20, frequency 40, is a bar 4 cm wide, 5 cm tall.
(a) Find the bar height for width 10, frequency 45.
(b) Find the frequency of a 3 cm by 3 cm bar.

Area of bar $\propto$ frequency: $4 \times 5 = 20$ cm^2 represents 40
so 1 unit of frequency $= 0.5$ cm^2
(a) Width $= 2$ cm, area $= 0.5 \times 45 = 22.5$ cm^2, height $= 22.5 \div 2 = 11.25$ cm
(b) Area $= 3 \times 3 = 9$ cm^2, frequency $= 9 \div 0.5 = 18$
Answer: (a) 11.25 cm (b) 18

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} -3 \\ -9 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} -4 \\ -2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $-3 + 0\lambda = -4 + 1\mu$
j: $-9 + 3\lambda = -2 + (-1)\mu$
Solving i and j: $\lambda = 2$, $\mu = 1$
k check: $1 + (-2)(2) = -3$ and $-1 + (-2)(1) = -3$, equal
Point: $\begin{pmatrix} -3 \\ -3 \\ -3 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-3} \\ \mathbf{-3} \\ \mathbf{-3} \end{pmatrix}$

10 Finding mean and standard deviation from two probabilities (All boards)

The masses (kg) of parcels are normally distributed.
P(X < 85) = 0.0668 and P(X > 110) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.500$ and $z_2 = 0.9998$
$85 = \mu + (-1.500)\sigma$ and $110 = \mu + 0.9998\sigma$
Subtract: $25 = 2.500\,\sigma$, so $\sigma = 10.0$
$\mu = 85 - (-1.500)(10.00) = 100$
Answer: $\mu = 100$, $\sigma = 10.0$