**Day 103** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of age of a house and its energy cost give $r = -0.795$.
Two-tailed critical value (5%): 0.7067.
Test for correlation.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
$|r| = 0.795 > 0.7067$
r is in the critical region, so reject $H_0$
Conclusion: there is evidence of correlation between age of a house and its energy cost
**Answer: reject $H_0$: there is evidence of correlation between age of a house and its energy cost**

## 2 Maximum speed from acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. At time t s its velocity is $v = 6t - 3t^2$ m s$^{-1}$, for $t \geq 0$.
Find the maximum speed of the particle and the time at which it occurs.

$a = dv/dt = 6 - 6t$
Maximum speed when $a = 0$: $t = 6 \div 6 = 1$
$v = 6 \times 1 - 3 \times 1^2$
**Answer: 3 m s$^{-1}$ at t = 1 s**

## 3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $3x^2 + 36x + 105$ in the form $a(x + p)^2 + q$.
(b) Write down the coordinates of the minimum point of $y = 3x^2 + 36x + 105$.

Take out the factor 3: $3x^2 + 36x + 105 = 3(x^2 + 12x) + 105$
$= 3[(x + 6)^2 - 36] + 105$
$= 3(x + 6)^2 - 3$
$(x + 6)^2 \geq 0$ with equality at $x = -6$, so the minimum is $-3$ at $x = -6$.
**Answer: (a) $3(x + 6)^2 - 3$ (b) $(-6, -3)$**

## 4 Scatter diagram (AQA, Edexcel, OCR, MEI, Eduqas)

The scatter diagram shows eight pairs of values of x and y.
(a) Describe the correlation.
(b) Why should $y = 4 + 1.5x$ not be used for $x = 15$?

(a) The points lie close to a straight line rising: strong positive correlation
(b) The data cover x = 1 to 10; x = 15 is outside this range
so using the line would be extrapolation and may be unreliable
**Answer: (a) strong positive correlation (b) extrapolation outside the range of the data**

## 5 Vectors in a triangle (proof and ratio) (All boards)

In triangle OAB, OA = a and OB = b. M lies on AB with AM : MB = 3 : 1, and N lies on OB with ON : NB = 1 : 1.
Find OM and NM in terms of a and b.

$AB = b - a$, so $AM = \frac{3}{4}(b - a)$

$OM = a + AM = \frac{a}{4} + \frac{3b}{4}$

$ON = \frac{b}{2}$, so $NM = OM - ON = \frac{a}{4} + \frac{b}{4}$

**Answer: $\mathbf{OM = \frac{a}{4} + \frac{3b}{4}}$, $\mathbf{NM = \frac{a}{4} + \frac{b}{4}}$**

## 6 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{12}{4-2\sqrt{6}}$ in the form $p + q\sqrt{6}$, where p and q are integers.

Multiply top and bottom by the conjugate $(4 + 2\sqrt{6})$:

Denominator: $(4-2\sqrt{6})(4+2\sqrt{6}) = 4^2 - 2^2\times 6 = -8$

Numerator: $(12)(4+2\sqrt{6}) = 48 + 24\sqrt{6}$

So the fraction = $\frac{48+24\sqrt{6}}{-8} = -6 - 3\sqrt{6}$

**Answer: $-6 - 3\sqrt{6}$**

## 7 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 3x - 12$. Use the Newton-Raphson method with $x_0 = 2$ to find $x_1$ and $x_2$, to 4 d.p.

$f'(x) = 3x^2 + 3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \frac{2.000}{15.00} = 1.8667$

$x_2 = 1.8667 - \frac{0.1043}{13.45} = 1.8589$

**Answer: $x_1 = 1.8667$, $x_2 = 1.8589$**

## 8 Using a binomial expansion to estimate (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1-x)^{\frac{1}{2}}$ up to the $x^2$ term.

(b) Use $x = 0.02$ to estimate $\sqrt{0.98}$ to 5 decimal places.

(a) $(1+ax)^n = 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$ with $n = \frac{1}{2}$, $a = -1$:

$= 1 - \frac{1}{2}x - \frac{1}{8}x^2$

(b) $1 - 1 \times 0.02 = 0.98$, so substitute $x = 0.02$:

$1 - \frac{1}{2}\times 0.02 - \frac{1}{8}\times 0.02^2 = 0.98995$

**Answer: (a) $1 - \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$ (b) 0.98995**

## 9 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train accelerates uniformly from rest at 1.5 m $s^{-2}$ for 6 s, then travels at constant speed for 20 s, and finally decelerates uniformly at 1 m $s^{-2}$ to rest.

Find the total distance travelled and the total time of the journey.

Max speed = $1.5 \times 6 = 9$

Stage 1: ½ × 6 × 9 = 27; stage 2: 9 × 20 = 180

Stage 3: time = 9 ÷ 1 = 9, distance = ½ × 9 × 9 = 40.5

**Answer: 247.5 m; 35 s**

## 10 Optimisation: open box and tank (All boards)

Squares of side x cm are cut from the corners of a square card of side 30 cm and the sides are folded up to make an open box.

Find x for the greatest volume, and that volume.

$V = x(30 - 2x)^2 = 4x^3 - 120x^2 + 900x$

$\frac{dV}{dx} = 12x^2 - 240x + 900 = (6x - 30)(2x - 30) = 0$

$x < 15$, so $x = 5$

$\frac{d^2V}{dx^2} = 24x - 240 = -120 < 0$ at $x = 5$, so maximum

$V = 5 \times 20^2 = 2000$

**Answer: $x = 5$, $V = 2000\ cm^3$**