**Day 102** **A Level Maths: B Tier 10-a-day: Model Solutions**

## 1 Completing the square proof (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Prove that $2x^2 + 12x + 23 > 0$ for all real values of x.

Complete the square: $2x^2 + 12x + 23 = 2(x + 3)^2 + 5$

$(x + 3)^2 \geq 0$ for all real x, so $2(x + 3)^2 \geq 0$.

Adding 5: the expression is $\geq 5$, so $> 0$ for all real x.

**Answer: $2x^2 + 12x + 23 > 0$ for all x**

## 2 Chain rule (All boards)

Given that $y = e^{3x - 3}$,

find $\frac{dy}{dx}$, and the gradient of the curve when x = 1.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function

$\frac{dy}{dx} = 3e^{3x - 3}$

When x = 1: gradient = 3

**Answer: $\frac{dy}{dx} = 3e^{3x - 3}$; gradient 3**

## 3 Frequency table: mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows the number of siblings, x, of 47 students.

Calculate the mean and standard deviation (3 s.f.).

$\Sigma f = 47$, $\Sigma fx = 99$, $\Sigma fx^2 = 305$

Mean $= \frac{99}{47} = 2.11$

Variance $= \frac{305}{47} - (2.1064)^2 = 2.053$

Standard deviation = 1.43

**Answer: mean 2.11, standard deviation 1.43**

## 4 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 6x - 2y + 6 = 0$.

Find the coordinates of its centre and its radius.

Complete the square in x and y:

$(x + 3)^2 - 9 + (y - 1)^2 - 1 + 6 = 0$

$(x + 3)^2 + (y - 1)^2 = 4$

Centre $(-3, 1)$, radius 2

**Answer: centre $(-3, 1)$, radius 2**

## 5 Pulling at an angle: smooth surface (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sledge of mass 6 kg is pulled across smooth horizontal ice by a rope with tension 26 N, inclined at $\alpha$ above the horizontal, where $\tan \alpha = \frac{5}{12}$.

Take $g = 9.8$ m s$^{-2}$.

Find (a) the acceleration of the sledge, (b) the normal reaction of the ice.

$\cos \alpha = 12/13$, $\sin \alpha = 5/13$

(a) Horizontal: $26 \times 12/13 = 6a$

(b) Vertical: $R + 26 \times 5/13 = 6 \times 9.8$

**Answer: (a) 4 m s$^{-2}$ (b) 48.8 N**

## 6 Trigonometric modelling (tides) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The height of the tide in a harbour is modelled by $H = 7 + 6\sin(\frac{\pi t}{12})$, where t is in hours after midnight.

Find (a) the greatest and least values, (b) the values of t with $0 \le t \le 24$ when it equals 4 m.

(a) sin ranges from $-1$ to 1: greatest $7 + 6 = 13$, least $7 - 6 = 1$

(b) $7 + 6\sin(\frac{\pi t}{12}) = 4$, so $\sin(\frac{\pi t}{12}) = -\frac{1}{2}$

$\frac{\pi t}{12} = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, so t = 14 or 22

**Answer: (a) 13 m, 1 m (b) t = 14 and 22**

## 7 Jumping to a different journey: two-stage motion (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A coach accelerates uniformly from rest at 0.5 m $s^{-2}$ for 6 s, then travels at constant speed for 20 s, and finally decelerates uniformly at 1.5 m $s^{-2}$ to rest.

Find the total distance travelled and the total time of the journey.

Max speed = $0.5 \times 6 = 3$

Stage 1: $\frac{1}{2} \times 6 \times 3 = 9$; stage 2: $3 \times 20 = 60$

Stage 3: time = $3 \div 1.5 = 2$, distance = $\frac{1}{2} \times 3 \times 2 = 3$

**Answer: 72 m; 28 s**

## 8 Binomial two-tailed test (All boards)

A coin is thought to land heads with probability 0.25. It is tossed 30 times and lands heads 3 times.

Test, at the 5% significance level, whether the probability of heads is different from 0.25.

$H_0$: $p = 0.25$, $H_1$: $p \neq 0.25$. Under $H_0$, $X \sim B(30, 0.25)$

$P(X \le 3) = 0.0374$

Two-tailed: compare with 0.025. $0.0374 > 0.025$, so do not reject $H_0$

There is insufficient evidence that the probability of heads is different from 0.25

**Answer: do not reject $H_0$: insufficient evidence that the probability of heads is different from 0.25**

## 9 Cubic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $(x - 4)(x - 3)(x - 1) \ge 0$.

Critical values: $x = 1$, $x = 3$, $x = 4$

The cubic has a positive $x^3$ term, so the graph goes from bottom left to top right and crosses the axis at each critical value.

Reading from the sketch: $1 \le x \le 3$ or $x \ge 4$

**Answer: $1 \le x \le 3$ or $x \ge 4$**

## 10 Optimisation: open box and tank (All boards)

An open tank has a square base of side x m and volume 108 $m^3$.

Show that its surface area is $S = x^2 + \frac{432}{x}$, and find the minimum value of S.

Height $h = \frac{108}{x^2}$, so $S = x^2 + 4xh = x^2 + \frac{432}{x}$

$\frac{dS}{dx} = 2x - \frac{432}{x^2} = 0$, so $x^3 = 216$ and $x = 6$

$\frac{d^2S}{dx^2} = 2 + \frac{864}{x^3} > 0$, so minimum

$S = 36 + 72 = 108$ $m^2$

**Answer: S = 108 $m^2$ (when x = 6)**