Day 101 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 3D vectors: magnitude, distance and unit vectors (All boards)

The vector $\mathbf{a} = \begin{pmatrix} -4 \\ -3 \\ 12 \end{pmatrix}$.

Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

$|\mathbf{a}| = \sqrt{4^2 + 3^2 + 12^2} = \sqrt{169} = 13$

Unit vector $= \frac{1}{13}\mathbf{a}$

Answer: $|\mathbf{a}| = 13$, unit vector $\frac{1}{13}\begin{pmatrix} -4 \\ -3 \\ 12 \end{pmatrix}$

2 Coded data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

7 values of x are coded using $y = \frac{(x - 100)}{5}$.

$\Sigma y = -4$ and $\Sigma y^2 = 66$.

Find the mean and standard deviation of x (3 s.f.).

Mean of $y = -4 \div 7 = -0.5714$

Mean of $x = 100 + 5 \times -0.5714 = 97.1$

Variance of $y = \frac{66}{7} - (-0.5714)^2 = 9.102$

SD of y = 3.017, so SD of $x = 5 \times 3.017 = 15.1$

Answer: mean 97.1, standard deviation 15.1

3 Speed-time graph: trapezium (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph of a train between two stations.

Find (a) the acceleration in the first 8 s, (b) the distance between the stations.

(a) a = gradient $= 10 \div 8$

(b) Distance = area of trapezium $= \frac{1}{2} \times (32 + 20) \times 10$

Answer: (a) 1.25 m s^{-2} (b) 260 m

4 Chord of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -7x - 35$ meets the circle $(x + 1)^2 + (y + 3)^2 = 125$ at the points A and B. Find the coordinates of A and B, and the exact length of the chord AB.

Substitute $y = -7x - 35$: $(x + 1)^2 + (-7x - 32)^2 = 125$

$x^2 + 9x + 18 = 0$

$(x + 3)(x + 6) = 0$, so $x = -6$ or $x = -3$

$y = -7x - 35$: A = (−6, 7), B = (−3, −14)

$AB^2 = 3^2 + (-21)^2 = 450$

Answer: A(−6, 7), B(−3, −14), AB = $15\sqrt{2}$

5 Will it move? Limiting friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A box of mass 5 kg rests on a rough horizontal floor, with coefficient of friction 0.4. A horizontal force of 24 N is applied to the box.

Take g = 9.8 m s^{-2}.

Determine whether the box moves. Find the friction force, or the acceleration if it moves.

$R = 5 \times 9.8 = 49$, maximum friction $\mu R = 0.4 \times 49 = 19.6$

$24 > 19.6$ so the box moves, $F = 19.6$

$24 - 19.6 = 5a$

Answer: It moves; a = 0.88 m s^{-2}

6 Factor theorem with an unknown (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = 2x^3 - 4x^2 - 22x + k$.
Given that $(x - 1)$ is a factor of $f(x)$,
(a) find the value of k,
(b) factorise $f(x)$ completely.

$f(1) = 0$: $2 - 4 - 22 + k = 0$
$k = 24$
$f(x) = (x - 1)(2x^2 - 2x - 24)$
$= 2(x - 4)(x - 1)(x + 3)$
Answer: (a) $k = 24$ (b) $f(x) = 2(x - 4)(x - 1)(x + 3)$

7 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{4}} \sin 2x \cos 2x \, dx$.

Use $\sin 2x \cos 2x = \frac{1}{2} \sin 4x$

$\int \sin 2x \cos 2x \, dx = -\frac{\cos 4x}{8}$

$= \left[-\frac{\cos 4x}{8}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}$

Answer: $\frac{1}{4}$

8 Coefficient in a product with a binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(2 + x)(3 - 3x)^4$.

$(3 - 3x)^4$ has x^1 coefficient -324 and x^2 coefficient 486
x^2 term of the product $= 2 \times 486 + 1 \times (-324)$
$= 648$
Answer: 648

9 Intersection of two lines in 3D (CIE)

Line l_1: $\mathbf{r} = \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$.

Line l_2: $\mathbf{r} = \begin{pmatrix} 4 \\ -2 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$.

Show that l_1 and l_2 intersect and find the point.

i: $2 + (-2)\lambda = 4 + 2\mu$
j: $-5 + 3\lambda = -2 + 0\mu$
Solving i and j: $\lambda = 1$, $\mu = -2$
k check: $-2 + (-1)(1) = -3$ and $-7 + (-2)(-2) = -3$, equal

Point: $\begin{pmatrix} 0 \\ -2 \\ -3 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{0} \\ \mathbf{-2} \\ \mathbf{-3} \end{pmatrix}$

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10 Finding mean and standard deviation from two probabilities (All boards)

The random variable X is normally distributed.
P(X < 56) = 0.0228 and P(X > 92) = 0.1587.
Find the mean and standard deviation (3 s.f.).

$z_1 = -1.999$ and $z_2 = 0.9998$
$56 = \mu + (-1.999)\sigma$ and $92 = \mu + 0.9998\sigma$
Subtract: $36 = 2.999\,\sigma$, so $\sigma = 12.0$
$\mu = 56 - (-1.999)(12.00) = 80.0$
Answer: $\mu = 80.0$, $\sigma = 12.0$