Day 100 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Binomial expected frequency (All boards)

Each of 7 people independently supports a team with probability 0.25; X is the number of supporters. This is done 100 times. Estimate the number of times that X = 1.

X ~ B(7, 0.25)

$P(X = 1) = {}^7C_1 \times (0.25)^1 \times (0.75)^6 = 0.3115$

Expected = 100 × 0.3115 = 31.15

Answer: about 31 times

2 Constant acceleration with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has velocity $(5\mathbf{i} + 6\mathbf{j})$ m s^{-1} at time t = 0 and moves with constant acceleration $(2\mathbf{i} + 3\mathbf{j})$ m s^{-2}. Find its velocity and speed when t = 2.

$\mathbf{v} = \mathbf{u} + \mathbf{at} = (5\mathbf{i} + 6\mathbf{j}) + 2(2\mathbf{i} + 3\mathbf{j})$

$= 9\mathbf{i} + 12\mathbf{j}$

Speed $= \sqrt{(9^2 + 12^2)}$

Answer: $\mathbf{v} = (9\mathbf{i} + 12\mathbf{j})$ m s^{-1}; speed 15 m s^{-1}

3 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-5 \le -x + 2 \le 5$.

Subtract 2 from each part: $-7 \le -x \le 3$

Divide each part by -1 (reversing the inequalities): $-3 \le x \le 7$

Answer: $-3 \le x \le 7$

4 Laws of logarithms: solving equations (All boards)

Solve $\log_2 x + \log_2(x + 6) = 4$.

$\log_2(x(x + 6)) = 4$, so $x(x + 6) = 2^4 = 16$

$x^2 + 6x - 16 = 0$, so $(x - 2)(x + 8) = 0$

$x > 0$ for the logs to exist, so $x = 2$

Answer: $x = 2$

5 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(1 + x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^3.

(b) State the range of values of x for which the expansion is valid.

$$(1 + ax)^n = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \ldots$$

Here $n = \frac{1}{3}$ and $a = 1$:

coefficients: x: $\frac{1}{3}$, x^2: $-\frac{1}{9}$, x^3: $\frac{5}{81}$

Valid when $|x| < 1$, that is $|x| < 1$

Answer: (a) $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 + \ldots$ (b) $|x| < 1$

6 Integration by substitution (given substitution) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^1 x(x^2 + 1)^4 \, dx$.

$\frac{du}{dx} = 2x$, so $x \, dx = \frac{1}{2} du$

Limits: x = 0 gives u = 1, x = 1 gives u = 2

$\int_1^2 \frac{1}{2}u^4 \, du = [\frac{u^5}{10}]_1^2$

$= \frac{31}{10}$

Answer: $\frac{31}{10}$

7 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -t$, $y = t^2 - 4t + 3$.
The line $y = 3x + 3$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $t^2 - 4t + 3 = -3t + 3$
$t^2 - t = 0$, so $(t - 1)t = 0$
$t = 0$ or $t = 1$, giving the points (0, 3) and (−1, 0)
Answer: (0, 3) and (−1, 0)

8 Area under a curve (All boards)

The region R is bounded by the axes, the line $x = \ln 2$
and the curve $y = 3e^{3x} + 1$. Find the exact area of R.

Area $= \int_0^{\ln 2} (3e^{3x} + 1)\,dx = [x + e^{3x}]_0^{\ln 2}$
$= \ln 2 + 7$
Answer: ln 2 + 7

9 Continuous random variable: median (CIE)

The continuous random variable X has pdf $f(x) = \frac{3x^2}{216}$ for $0 \le x \le 6$, and 0 otherwise.

(a) Find P(X > 1).
(b) Find the median of X, to 3 significant figures.

$F(x) = \int_0^x f(t)\,dt = \frac{x^3}{216}$

(a) $P(X > 1) = 1 - F(1) = \frac{215}{216}$

(b) $F(m) = 0.5$: $\frac{m^3}{216} = \frac{1}{2}$

$m = 4.76$

Answer: (a) $\frac{215}{216}$ (b) 4.76

10 Rough table and a hanging particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 7 kg lies on a rough horizontal table (coefficient of friction 0.5). It is joined by a light string over a smooth pulley to a particle of mass 8 kg hanging freely.

Take $g = 9.8$ m s^{-2}.

The system is released. Find the acceleration and the tension in the string.

Block: $T - 0.5 \times 7g = 7a$
Hanging: $8g - T = 8a$
Add: $8g - 3.5g = 15a$
$a = 2.94$
$T = 8(9.8 - 2.94)$

Answer: a = 2.94 m s^{-2}; T = 54.88 N