Day 10 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=4}^{12}(4r - 3)$.

There are $12 - 4 + 1 = 9$ terms
First term = 13, last term = 45
$S = \frac{9}{2}(13 + 45) = 261$

Answer: 261

2 Chain rule (All boards)

Given that $y = (2x^2 + 3)^4$,
find $\frac{dy}{dx}$, and the gradient of the curve when $x = -1$.

Chain rule: differentiate the outer function, then multiply by the derivative of the inner function
$\frac{dy}{dx} = 16x(2x^2 + 3)^3$
When $x = -1$: gradient $= -2000$

Answer: $\frac{dy}{dx} = 16x(2x^2 + 3)^3$; gradient -2000

3 Vertical projection upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is projected vertically upwards from ground level with speed 29.4 m s^{-1}.
Take g = 9.8 m s^{-2}.
Find (a) the greatest height reached, (b) the total time before it returns to the ground.

(a) At the top $v = 0$: $0 = 29.4^2 - 2 \times 9.8 \times H$
$H = 864.36 \div 19.6 = 44.1$
(b) $s = 0$: $0 = 29.4\,t - 4.9t^2$, so $t = 29.4 \div 4.9$
Answer: (a) 44.1 m (b) 6 s

4 Large data set: outliers (AQA, Edexcel, OCR, MEI, Eduqas)

Large data set: mean windspeed (knots) at Leeming has mean 8, sd 4.
Outliers are more than 3 sd from the mean.
Are the values 17 and −6 outliers?

Limits: $8 \pm 3 \times 4$, so from -4 to 20
17 is inside the limits, so it is not an outlier
-6 is outside the limits, so it is an outlier
Answer: 17: not an outlier; −6: outlier

5 Power, driving force and resistance (CIE)

A car of mass 1500 kg moves along a horizontal road. The engine works at a constant rate of 50 kW. The resistance to motion is constant at 500 N.
Find the acceleration when the speed is 10 m s^{-1}.

$P = Fv$: $F = 50000 \div 10 = 5000$ N
$F - R = ma$: $5000 - 500 = 1500\,a$
Answer: 3 m s^{-2}

6 Total probability and Bayes (All boards)

R: it rains; L: Ali is late.
P(R) = 0.25, P(L | R) = 0.6, P(L | R') = 0.2.
(a) Find P(L).
(b) Find P(R | L).

(a) $P(L) = 0.25 \times 0.6 + 0.75 \times 0.2 = 0.15 + 0.15 = 0.3$

(b) $P(R \mid L) = \frac{0.25 \times 0.6}{0.3} = \frac{0.15}{0.3} = 0.500$

Answer: (a) 0.3 (b) $\frac{1}{2}$

7 Tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle C has equation $(x + 4)^2 + (y - 4)^2 = 10$.
The point P(−3, 7) lies on C.
Find an equation of the tangent to C at P, in the form ax + by + c = 0 with integer a, b, c.

Centre of C = (−4, 4). Gradient of radius CP = $\frac{3}{1} = 3$

The tangent is perpendicular to the radius: gradient = $-\frac{1}{3}$

$y - 7 = -\frac{1}{3}(x + 3)$

Multiply out and collect terms: $x + 3y - 18 = 0$

Answer: $x + 3y - 18 = 0$

8 Reduction to linear form (logarithms) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The variables x and y satisfy $y = ax^n$. A graph of ln y against ln x is a straight line with gradient 2 that passes through (0, ln 5).
Find the values of a and n.

$\ln y = \ln a + n \ln x$
gradient = n = 2
intercept $\ln a = \ln 5$, so a = 5

Answer: a = 5, n = 2

9 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence has 9th term −31 and 13th term −55.
(a) Find the first term and the common difference.
(b) Find the sum of the first 30 terms.

$a + 8d = -31$ and $a + 12d = -55$
Subtract: $4d = -24$, so $d = -6$ and $a = 17$

$S_{30} = \frac{30}{2}(2 \times 17 + 29 \times (-6)) = -2100$

Answer: (a) a = 17, d = −6 (b) −2100

10 Integrating trigonometric functions using identities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the exact value of

$\int_0^{\frac{\pi}{6}} \cos^2 3x \, dx$.

Use $\cos^2 3x = \frac{1}{2}(1 + \cos 6x)$

$\int \cos^2 3x \, dx = \frac{x}{2} + \frac{\sin 6x}{12}$

$= \left[\frac{x}{2} + \frac{\sin 6x}{12}\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{12}$

Answer: $\frac{\pi}{12}$