Day 1 **A Level Maths: B Tier 10-a-day: Model Solutions**

1 Resultant force and acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(\mathbf{i} - 2\mathbf{j})$ N and $(7\mathbf{i} + 8\mathbf{j})$ N act on a particle of mass 5 kg.
Find the magnitude of the acceleration of the particle.

Resultant = $(\mathbf{i} - 2\mathbf{j}) + (7\mathbf{i} + 8\mathbf{j}) = 8\mathbf{i} + 6\mathbf{j}$
$|R| = \sqrt{(8^2 + 6^2)} = 10$ N
$a = F/m = 10 \div 5$
Answer: 2 m s^{-2}

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

Items numbered 1 to 180; systematic sample of size 15.
(a) Find the sampling interval.
(b) The first item is number 4. Find the 4-th item.

(a) Interval = $180 \div 15 = 12$
(b) 4-th item = $4 + 3 \times 12 = 40$
Answer: interval 12; item number 40

3 Arithmetic sequences and series (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The first term of an arithmetic series is 4 and the common difference is 3.
The sum of the first n terms is 781. Find the value of n.

$S_n = \frac{n}{2}(2 \times 4 + (n - 1) \times 3) = 781$

$3n^2 + 5n - 1562 = 0$
$(n - 22)(3n + 71) = 0$
n must be positive, so n = 22
Answer: n = 22

4 Inverse normal (All boards)

A battery lasts X hours, normal with mean 100, s.d. 8.
Find k (3 s.f.): (a) $P(X < k) = 0.1$,
(b) $P(X > k) = 0.05$.

(a) $z = -1.282$, so $k = 100 + (-1.282) \times 8 = 89.7$
(b) $P(X < k) = 0.95$, $z = 1.645$, so $k = 100 + 1.645 \times 8 = 113$
Answer: (a) 89.7 (b) 113

5 Equations with sec, cosec and cot (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2\tan^2 x - 5\sec x + 5 = 0$ for $0 \leq x \leq 360°$,
giving answers to 1 d.p.

Use $\tan^2 x = \sec^2 x - 1$: $2\sec^2 x - 5\sec x + 3 = 0$
$(\sec x - 1)(2\sec x - 3) = 0$, so $\sec x = 1$ or $\frac{3}{2}$
$\sec x = 1$, so $\cos x = 1$: $x =$
$\sec x = \frac{3}{2}$, so $\cos x = \frac{2}{3}$: $x = 48.2°, 311.8°$
Answer: x = 48.2°, 311.8°

6 Inclined plane: force to hold or move the particle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A block of mass 4 kg is held on a rough plane ($\tan\alpha = \frac{3}{4}$, $\mu = 0.25$) by a force P N up the plane, as shown.
Take g = 9.8 m s^{-2}. The block is about to move up the plane. Find P.

$R = mg\cos\alpha = 39.2 \times 4/5 = 31.36$
Friction (limiting) $F = \mu R = 7.84$
Moving up, friction acts down: $P = mg\sin\alpha + F$
$mg\sin\alpha = 23.52$
Answer: 31.36 N

7 Binomial expansion (any power) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Expand $(9 - 4x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3.
(b) State the range of values of x for which the expansion is valid.

Take out 9: $(9 - 4x)^{\frac{3}{2}} = 9^{\frac{3}{2}}(1 + -\frac{4}{9}x)^{\frac{3}{2}} = 27(1 + -\frac{4}{9}x)^{\frac{3}{2}}$

Expand $(1 + -\frac{4}{9}x)^{\frac{3}{2}} = 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + ...$

Multiplying by 27: coefficients of 1, x, x^2, x^3 are 27, -18, 2, $\frac{4}{27}$

Valid when $|\frac{4}{9}x| < 1$, that is $|x| < \frac{9}{4}$

Answer: (a) $27 - 18x + 2x^2 + \frac{4}{27}x^3 + ...$ (b) $|x| < \frac{9}{4}$

8 Partial fractions and integration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

(a) Write $\frac{2x - 1}{(x + 1)(x + 4)}$ in partial fractions.

(b) Hence evaluate $\int_0^2 \frac{2x - 1}{(x + 1)(x + 4)}\,dx$,
giving the answer as a single logarithm.

(a) $\frac{2x - 1}{(x + 1)(x + 4)} \equiv \frac{A}{x + 1} + \frac{B}{x + 4}$, so $2x - 1 \equiv A(x + 4) + B(x + 1)$
$x = -1$: $A = -1$; $x = -4$: $B = 3$

(b) $\int_0^2 (\frac{-1}{x + 1} + \frac{3}{x + 4})\,dx = [-\ln(x + 1) + 3\ln(x + 4)]_0^2$

$= -\ln\frac{3}{1} + 3\ln\frac{6}{4} = \ln(\frac{9}{8})$

Answer: (a) $A = -1$, $B = 3$ (b) $\ln(\frac{9}{8})$

9 Parametric curve meeting a line (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2t + 3$, $y = -t^2 - 6t - 3$.
The line $y = -x + 3$ meets the curve at two points.
Find the coordinates of these points.

Substitute x and y into the line: $-t^2 - 6t - 3 = -2t$
$-t^2 - 4t - 3 = 0$, so $-(t + 1)(t + 3) = 0$
$t = -3$ or $t = -1$, giving the points $(-3, 6)$ and $(1, 2)$
Answer: $(-3, 6)$ and $(1, 2)$

10 Sector and arc in terms of π (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sector of a circle has radius 10 cm and angle 30°.
Find, in terms of π, the arc length and the area of the sector.

$\theta = 30° = \frac{\pi}{6}$ radians

Arc length $= r\theta = \frac{5\pi}{3}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{25\pi}{3}$ cm^2

Answer: arc $\frac{5\pi}{3}$ cm, area $\frac{25\pi}{3}$ cm^2