Day 99 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $3^{x+3} = 5^{2x}$.
Give your answer to 3 significant figures.

Take logs: $(x + 3) \ln 3 = 2x \ln 5$
Collect x terms: $x(\ln 3 - 2 \ln 5) = -3 \ln 3$
$x = \dfrac{-3 \ln 3}{\ln 3 - 2 \ln 5}$
$x = 1.55$
Answer: $x = 1.55$

2 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−14, 3) and B(10, −7) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−2, −15) lies on the circle.

Centre = midpoint of AB = (−2, −2)
$r^2 = (-12)^2 + (5)^2 = 169$
$(x + 2)^2 + (y + 2)^2 = 169$: $x^2 + y^2 + 4x + 4y - 161 = 0$
(b) $CM^2 = (0)^2 + (-13)^2 = 169 = r^2$
Answer: (a) $x^2 + y^2 + 4x + 4y - 161 = 0$

3 Exact values from the addition formulae (All boards)

By writing 195° as 150° + 45°, show that
$\tan 195° = 2 - \sqrt{3}$.

$\tan 195° = \tan(150° + 45°) = \dfrac{\tan 150° + \tan 45°}{1 - \tan 150° \tan 45°}$

$\tan 30° = \dfrac{\sqrt{3}}{3}$, $\tan 45° = 1$, $\tan 60° = \sqrt{3}$, $\tan 120° = -\sqrt{3}$, $\tan 150° = -\dfrac{\sqrt{3}}{3}$

$= \dfrac{(-\frac{\sqrt{3}}{3}) + 1}{1 - (-\frac{\sqrt{3}}{3}) \times 1}$, which rationalises to $2 - \sqrt{3}$

Answer: $2 - \sqrt{3}$

4 Foot of a perpendicular (All boards)

The line l has equation $x + 2y = -8$.
(a) Find the coordinates of the foot of the perpendicular from P(4, −2) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = $-\dfrac{1}{2}$, so perpendicular gradient = 2
Perpendicular through P: $2x - y = 10$
Solve with l: $x = \dfrac{12}{5}$, $y = -\dfrac{26}{5}$
$\text{Distance}^2 = (4 - \frac{12}{5})^2 + (-2 - (-\frac{26}{5}))^2 = \dfrac{64}{5}$
Answer: (a) $(\frac{12}{5}, -\frac{26}{5})$ (b) $\frac{8\sqrt{5}}{5}$

5 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (4, 0, 0), (5, 3, 2) and (−3, −1, −2).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix}-1\\-3\\-2\end{pmatrix}$, BC = $\begin{pmatrix}-8\\-4\\-4\end{pmatrix}$

BA·BC = 28

|BA| = $\sqrt{14}$, |BC| = $\sqrt{96}$

cos ABC = $\frac{28}{\sqrt{14}\times\sqrt{96}}$ = 0.7638, so angle ABC = 40.2°

Answer: 40.2°

6 Power, driving force and resistance (CIE)

A 1500 kg car has engine power 80 kW and constant resistance 500 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 12 m s^{-1} up a hill where sin α = $\frac{1}{20}$. Take g = 10. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. P = Fv: 80000 = 500 v

(b) Driving force F = P ÷ v = 80000 ÷ 12 = $\frac{20000}{3}$ N

Up the hill: F − 500 − 1500 g sin α = 1500 a

$\frac{20000}{3}$ − 500 − 750 = 1500 a

a = $\frac{65}{18}$

Answer: (a) 160 m s^{-1} (b) 3.61 m s^{-2}

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^3 in the expansion of
$(2x^3 - \frac{3}{x^2})^6$.

General term: $^6C_r\,(2x^3)^{6-r}\,(-3x^{-2})^r$

Power of x: 3(6 − r) − 2r = 3

18 − 5r = 3 so r = 3

Coefficient = $^6C_3 \times 2^3 \times (-3)^3 = -4320$

Answer: −4320

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf f(x) = $\frac{3}{216}x^2$ for 0 ≤ x ≤ 6, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for 0 ≤ x ≤ 6.
(b) Find P(X > 3). (c) Find the median of X to 3 s.f.

(a) F(x) = $\int_0^x$ f(t) dt = $\frac{x^3}{216}$

(b) P(X > 3) = 1 − F(3) = $\frac{7}{8}$

(c) F(m) = $\frac{1}{2}$: $m^3 = \frac{6^3}{2}$, so m = 6 × $(0.5)^{\frac{1}{3}}$

m = 4.76

Answer: (b) $\frac{7}{8}$ (c) 4.76

9 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{8}{15}$, $\mu = \frac{1}{3}$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{8}{17}$, $\cos\alpha = \frac{15}{17}$, weight W = 29.4 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 4.99$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 31.0$

Answer: least 4.99 N, greatest 31.0 N

10 Normal approximation to the Poisson (CIE)

X ~ Po(16). Use a suitable approximation to find $P(X \geq 12)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(16, 16) approximately

Continuity correction: P(Y > 11.5)

$z = -1.12$, so probability = 0.870

Answer: 0.870