Day 98	A Level Maths: A Tier 10-a-day
1 A particle of mass 4 kg is acted on by forces $F_1 = (-4\mathbf{i} + 11\mathbf{j})$ N and F_2. Its acceleration is $(\mathbf{i} - \mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.	
2 $f(x) = e^x - 2x - 5$. Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.	
3 The curve $y = f(x)$ has a minimum point at $(1, -2)$. Find the coordinates of the corresponding point on: (a) $y = f(x + 4)$ (b) $y = f(-x)$ (c) $y = f(\frac{x}{2})$	
4 A lift of mass 300 kg carries a person of mass 90 kg. The cable tension is 4056 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.	
5 $f(x) = \frac{-x - 3}{4x - 1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{4}$. Find $f^{-1}(x)$ and state its domain.	

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6 £20000 is invested at 6% compound interest per year.
Find the smallest whole number of years for the value to exceed £40000.

7 3% of people have a fault. The inspection test is positive for 90% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

8 $l_1: \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix}$

$l_2: \mathbf{r} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

9 $f(x) = x^2 - 2x + 6$, $x \in \mathbb{R}$, and $g(x) = \frac{2}{x+1}$, $x > -1$.
(a) Find the range of f.
(b) Show that gf is defined and find its range.

10 X, the mass (g), is $N(60, 5^2)$. The highest 30% of values exceed k.
Find k to 3 significant figures.