Day 98 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $F_1 = (-4\mathbf{i} + 11\mathbf{j})$ N and F_2. Its acceleration is $(\mathbf{i} - \mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 4(\mathbf{i} - \mathbf{j}) = 4\mathbf{i} - 4\mathbf{j}$

(a) $F_2 = (4\mathbf{i} - 4\mathbf{j}) - (-4\mathbf{i} + 11\mathbf{j}) = 8\mathbf{i} - 15\mathbf{j}$

(b) $|F_2| = \sqrt{(8^2 + -15^2)} = 17$

$\cos\theta = 8 \div 17$, so $\theta = 61.9°$

Answer: (a) $F_2 = (8\mathbf{i} - 15\mathbf{j})$ N (b) 61.9°

2 Change of sign to locate a root (All boards)

$f(x) = e^x - 2x - 5$.

Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

$f(x)$ is continuous on the interval

$f(2) = -1.61 < 0$

$f(3) = 9.09 > 0$

A change of sign, so a root lies in $[2, 3]$

Answer: Sign change: $f(2) = -1.61$, $f(3) = 9.09$

3 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a minimum point at $(1, -2)$.

Find the coordinates of the corresponding point on:

(a) $y = f(x + 4)$

(b) $y = f(-x)$

(c) $y = f(\frac{x}{2})$

(a) $y = f(x + 4)$: $(-3, -2)$

(b) $y = f(-x)$: $(-1, -2)$

(c) $y = f(\frac{x}{2})$: $(2, -2)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) $(-3, -2)$; (b) $(-1, -2)$; (c) $(2, -2)$

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 300 kg carries a person of mass 90 kg. The cable tension is 4056 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 390 kg), taking upwards as positive: $T - 390\,g = 390\,a$

$4056 - 3822 = 390\,a$, so $a = 0.6$

(a) a is positive, so the acceleration is 0.6 m s^{-2} upwards

(b) Person: $R - 90\,g = 90\,a$, so $R = 90(9.8 + 0.6)$

Answer: (a) 0.6 m s^{-2} upwards (b) 936 N

5 Inverse of a rational function (All boards)

$f(x) = \frac{-x - 3}{4x - 1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{4}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{-x - 3}{4x - 1}$: $y(4x - 1) = -x - 3$

$4xy - y = -x - 3$

$x(4y + 1) = y - 3$

$x = \frac{y - 3}{4y + 1}$, then swap x and y

Answer: $f^{-1}(x) = \frac{x - 3}{4x + 1}$, $x \neq -\frac{1}{4}$

6 Smallest whole number solving an exponential inequality (All boards)

£20000 is invested at 6% compound interest per year.
Find the smallest whole number of years for the value to exceed £40000.

$20000 \times 1.06^n > 40000$, so $1.06^n > 2$

$n \ln 1.06 > \ln 2$, so $n > \dfrac{\ln 2}{\ln 1.06} = 11.90$

$n = 12$

Answer: n = 12 years

7 Bayes' theorem in context (All boards)

3% of people have a fault. The inspection test is positive for 90% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) $P(+) = 0.03 \times 0.9 + 0.97 \times 0.02$
$= 0.027 + 0.0194 = 0.0464$

(b) $P(\text{has it} \mid +) = \dfrac{0.03 \times 0.9}{0.0464}$

$= 0.582$

Answer: (a) 0.0464 (b) 0.582

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}0\\2\\8\end{pmatrix} + \lambda\begin{pmatrix}-3\\1\\-3\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}-4\\1\\3\end{pmatrix} + \mu\begin{pmatrix}1\\2\\2\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $0 - 3\lambda = -4 + 1\mu$; $2 + 1\lambda = 1 + 2\mu$; $8 - 3\lambda = 3 + 2\mu$
Solving: $\lambda = 1$, $\mu = 1$, and all three equations are satisfied
Point of intersection = $(-3, 3, 5)$

Answer: Intersect at $(-3, 3, 5)$

9 Range of a composite function (All boards)

$f(x) = x^2 - 2x + 6$, $x \in \mathbb{R}$, and $g(x) = \dfrac{2}{x+1}$, $x > -1$.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x-1)^2 + 5 \geq 5$
So $f(x) \geq 5 > -1$, which lies in the domain of g, so gf is defined.

$gf(x) = \dfrac{2}{f(x)+1}$, with $f(x) \geq 5$, so $f(x) + 1 \geq 6$

Largest value is $\dfrac{2}{6} = \dfrac{1}{3}$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 5$ (b) $0 < gf(x) \leq \dfrac{1}{3}$

10 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is $N(60, 5^2)$. The highest 30% of values exceed k.
Find k to 3 significant figures.

$P(X > k) = 0.3$, so $P(X < k) = 0.7$
$z = \Phi^{-1}(0.7) = 0.524$
$k = 60 + 0.524 \times 5 = 62.6$

Answer: 62.6