Day 97 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean temperature (°C) in the same month are:
Perth: mean 15.5, standard deviation 3.1;
Jacksonville: mean 28, standard deviation 5.5.
Make two comparisons.

Means: Jacksonville has the higher mean (28 against 15.5), so on average the mean temperature is greater at Jacksonville
Standard deviations: Jacksonville is larger at Jacksonville (5.5 against 3.1)
so the readings at Jacksonville are more variable and those at Perth are more consistent
Answer: Jacksonville higher on average; Jacksonville more variable

2 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 20 m s^{-1} from a point 44.1 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $44.1 = \frac{1}{2} \times 9.8 \times t^2$, so t = 3
(a) Horizontal distance = 20 × 3
(b) Vertical speed on landing = 9.8×3 = 29.4; horizontal speed = 20
Speed = $\sqrt{(20^2 + 29.4^2)} = 35.6$
$\tan \theta = 29.4 \div 20$, $\theta = 55.8°$
Answer: (a) 60 m (b) 35.6 m s^{-1} at 55.8° below the horizontal

3 Logarithmic equations (All boards)

Solve $2\log_2 x - \log_2(x + 24) = 1$.

$\log_2 \frac{x^2}{x + 24} = 1$, so $\frac{x^2}{x + 24} = 2^1 = 2$
$x^2 = 2(x + 24)$: $x^2 - 2x - 48 = 0$
x = 8 or x = −6; $\log_2 x$ needs x > 0, so reject the negative root
Answer: x = 8

4 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t^2 + 4$, $y = 2t(t^2 - 1)$.
Find the coordinates of the points where the curve meets the x-axis.

y = 0: t = 0 or $t^2 = 1$, so t = 0, 1, −1
t = 0: x = 4
t = ±1: x = 5
Answer: (4, 0) and (5, 0)

5 Power, driving force and resistance (CIE)

A 1000 kg car has engine power 45 kW and constant resistance 1000 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 24 m s^{-1} up a hill where $\sin \alpha = \frac{1}{20}$. Take g = 10.

(a) Steady speed: driving force = resistance. P = Fv: 45000 = 1000 v
(b) Driving force F = P ÷ v = 45000 ÷ 24 = 1875 N
Up the hill: $F - 1000 - 1000 g \sin \alpha = 1000 a$
1875 − 1000 − 500 = 1000 a
a = 0.375
Answer: (a) 45 m s^{-1} (b) 0.375 m s^{-2}

6 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 4x + 6y - 87 = 0$ passes through P(6, −9).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−2, −3). Gradient of radius CP = $-\frac{3}{4}$

Gradient of tangent = $\frac{4}{3}$

$y + 9 = \frac{4}{3}(x - 6)$: $4x - 3y = 51$

$y = 0$: $x = \frac{51}{4}$

Answer: (a) $4x - 3y = 51$ (b) $(\frac{51}{4}, 0)$

7 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2 x + 6\cot x + 2 = 0$
for $-180° < x < 180°$, $x \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $2\cot^2 x + 6\cot x + 4 = 0$
$\cot x = -1$ or $\cot x = -2$
$\cot x = -1$ means $\tan x = -1$: $x = -45.0°, 135.0°$
$\cot x = -2$ means $\tan x = -\frac{1}{2}$: $x = -26.6°, 153.4°$

Answer: $x = -45.0°, -26.6°, 135.0°, 153.4°$

8 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(67, 5^2)$. Find $P(X > 73 \mid X > 68)$ to 3 significant figures.

Since $X > 73$ implies $X > 68$: $P = \frac{P(X > 73)}{P(X > 68)}$
$P(X > 73) = P(Z > 1.20) = 0.115$
$P(X > 68) = P(Z > 0.200) = 0.421$
$P = \frac{0.115}{0.421} = 0.273$

Answer: 0.273

9 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ k \\ 3 \end{pmatrix}$
relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$\text{AB} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$, and $\text{AC} = \begin{pmatrix} 0 \\ k - 3 \\ 4 \end{pmatrix}$

$\text{AC} = \lambda\text{AB}$: component 2 gives $4 = 3\lambda$, so $\lambda = \frac{4}{3}$

Hence $k = 3 + \frac{4}{3} \times 3 = 7$

$\text{AC} = \frac{4}{3}\text{AB}$, so $\text{BC} = \frac{1}{3}\text{AB}$: AB : BC = 3 : 1

Answer: $k = 7$, AB : BC = 3 : 1

10 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $2^{2x} - 9 \times 2^x + 8 = 0$.

Let $y = 2^x$. Then $y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ or $y = 1$

$2^x = 8$ gives $x = 3$; $2^x = 1$ gives $x = 0$

Answer: $x = 0$ or $x = 3$