Day 96 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Poisson probabilities (CIE)

Calls reach a switchboard at a mean rate of 1.5 per hour, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 2 (b) at least 2 (c) fewer than 2 (3 s.f.).

$X \sim Po(1.5)$

(a) $P(X = 2) = e^{-1.5}\ 1.5^2/2! = 0.251$
(b) $1 - P(X \leq 1) = 1 - 0.558 = 0.442$
(c) $P(X \leq 1) = 0.558$
Answer: (a) 0.251 (b) 0.442 (c) 0.558

2 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The radius of a sphere increases at 3 cm s^{-1}.
Find the rate of increase of its volume when r = 6 cm, in terms of π.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt} = 4\pi \times 6^2 \times 3 = 432\pi$

Answer: 432π cm^3/s

3 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 1$ and $g(x) = 3x + 2$.
Solve $fg(x) = gf(x)$.

$fg(x) = (3x + 2)^2 + 1 = 9x^2 + 12x + 5$
$gf(x) = 3(x^2 + 1) + 2 = 3x^2 + 5$
$9x^2 + 12x + 5 = 3x^2 + 5$ so $6x^2 + 12x = 0$
Answer: $x = -2$ or $x = 0$

4 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is $N(63, 5^2)$.
Find the symmetric limits about the mean between which the central 80% of values lie (3 s.f.).

Each tail has area 0.1, so $P(Z < z) = 0.9$
z = 1.28
Limits: $63 \pm 1.28 \times 5$
= 56.6 and 69.4
Answer: 56.6 to 69.4

5 Exact values from the addition formulae (All boards)

By writing 195° as 150° + 45°, find the exact value of

cos 195°, in the form $\frac{\sqrt{a} \pm \sqrt{b}}{c}$.

$\cos 195° = \cos(150° + 45°) = \cos 150° \cos 45° - \sin 150° \sin 45°$
$= (-\frac{\sqrt{3}}{2}) \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$
$= \frac{-\sqrt{2} - \sqrt{6}}{4}$
Answer: $\frac{-\sqrt{2} - \sqrt{6}}{4}$

6 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{5}{12}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{3}{4}$. Find the tension and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\theta = 0.6$, $\cos\theta = 0.8$

Along the plane: $T\cos\theta = 4g\sin\alpha$, so $T = 18.8$

Perpendicular: $R + T\sin\theta = 4g\cos\alpha$

$R = 4g\cos\alpha - T\sin\theta = 36.2 - 18.8\times0.600$

Answer: T = 18.8 N, R = 24.9 N

7 Range of a composite function (All boards)

$f(x) = x^2 - 2x + 6$, $x \in \mathbb{R}$, and $g(x) = \frac{8}{x-1}$, $x > 1$.

(a) Find the range of f.

(b) Show that gf is defined and find its range.

$f(x) = (x-1)^2 + 5 \geq 5$

So $f(x) \geq 5 > 1$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{8}{f(x)-1}$, with $f(x) \geq 5$, so $f(x) - 1 \geq 4$

Largest value is $\frac{8}{4} = 2$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 5$ (b) $0 < gf(x) \leq 2$

8 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 92.5 m above the sea at 10 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 8, vertical component 6 (upwards)

(a) $-92.5 = 6t - 4.9t^2$, so $4.9t^2 - 6t - 92.5 = 0$

$t = [6 + \sqrt{1849}] \div 9.8 = 5$ (taking the positive root)

(b) Distance = 8×5

(c) Vertical speed = $6 - 9.8\times5 = -43$; speed = $\sqrt{(8^2 + 43^2)}$

Answer: (a) 5 s (b) 40 m (c) 43.7 m s^{-1}

9 Differentiating sec, cosec, tan and cot (All boards)

$y = x\sec(x)$.

Find the exact gradient at $x = \frac{\pi}{3}$.

Product rule: $\frac{dy}{dx} = \sec(x) + x(\sec(x)\tan(x))$

At $x = \frac{\pi}{3}$: $\sec(x) = 2$ and the derivative term $= 2\sqrt{3}$

Gradient $= 2 + \frac{\pi}{3} \times 2\sqrt{3} = \frac{2\sqrt{3}\pi + 6}{3}$

Answer: gradient $\frac{2\sqrt{3}\pi + 6}{3}$

10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use x = 0.02 to estimate $\sqrt{96}$ to 3 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -2x$

(a) $1 - x - \frac{1}{2}x^2$

$x = \frac{1}{50}$: $96 = 10^2(1 - \frac{1}{25})$ so $\sqrt{96} = 10 \times (1 - \frac{1}{25})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{50} - \frac{1}{5000}) = 9.798000$

Answer: (a) $1 - x - \frac{1}{2}x^2$ (b) 9.798