**Day 95** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A club has 288 members: 36 Bronze, 156 Silver and 96 Gold.
A stratified sample of size 24 is taken.
Find the number of members sampled from each group.

Sampling fraction = $\frac{24}{288} = \frac{1}{12}$

Bronze: $36 \times \frac{1}{12} = 3$

Silver: $156 \times \frac{1}{12} = 13$

Gold: $96 \times \frac{1}{12} = 8$

**Answer: Bronze 3, Silver 13, Gold 8**

## 2 Polynomial division (All boards)

Find the quotient and the remainder when $x^4 + 3x^3 + x^2 - 10x - 9$ is divided by $(x^2 - 3)$.

Quotient (matching leading terms in turn): $x^2 + 3x + 4$
$(x^2 - 3)(x^2 + 3x + 4) = x^4 + 3x^3 + x^2 - 9x - 12$
Remainder = $x^4 + 3x^3 + x^2 - 10x - 9 - (x^4 + 3x^3 + x^2 - 9x - 12) = -x + 3$
**Answer: quotient $x^2 + 3x + 4$, remainder $-x + 3$**

## 3 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (7 kg) and B (1 kg) hang on a light string over a smooth pulley, A being 2.5 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $7g - T = 7a$; B: $T - 1g = 1a$
$a = 6g \div 8 = 7.35$, so $T = 1(g + a) = 17.15$
When A lands, B has risen 2.5 m with $v^2 = 2 \times 7.35 \times 2.5 = 36.75$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = 1.875$
Total = 2.5 + 1.875
**Answer: (a) 17.15 N (b) 4.375 m**

## 4 Double angle equations (All boards)

Solve $\cos 2\theta + \cos\theta = 0$
for $0° \le \theta \le 360°$.

$\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta + \cos\theta - 1 = 0$

Solve the quadratic: $\cos\theta = -1$ or $\cos\theta = \frac{1}{2}$

$\cos\theta = -1$: $\theta = 180°$

$\cos\theta = \frac{1}{2}$: $\theta = 60°, 300°$

**Answer: $\theta$ = 60°, 180°, 300°**

## 5 Foot of a perpendicular (All boards)

The line l has equation $x - y = 5$.
(a) Find the coordinates of the foot of the perpendicular from P(−4, 3) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 1, so perpendicular gradient = −1
Perpendicular through P: $-x - y = 1$
Solve with l: $x = 2$, $y = -3$
Distance$^2 = (-4 - 2)^2 + (3 - (-3))^2 = 72$
**Answer: (a) (2, −3) (b) $6\sqrt{2}$**

## 6 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t^2 - 1$, $y = t^3 - 3t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -2$, in the form $ax + by + c = 0$ (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{3t^2 - 3}{2t}$

$t = -2$: point $(3, -2)$, gradient $-\frac{9}{4}$

Normal gradient $= -1 \div (-\frac{9}{4}) = \frac{4}{9}$, so normal: $4x - 9y - 30 = 0$

**Answer: $\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$; $4x - 9y - 30 = 0$**

## 7 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg box is pulled from rest along rough horizontal ground by a horizontal force of 17.7 N and moves 7.5 m in 5 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = ½at^2$: $7.5 = ½a \times 5^2$, so $a = 0.6$

(a) $P - \mu 5g = 5a$: $17.7 - 49\mu = 3$, so $\mu = 0.3$

(b) $v = at = 0.6 \times 5 = 3$; after removal deceleration $= \mu g = 2.94$

$0 = 3^2 - 2 \times 2.94 \times s$, $s = 1.53$

**Answer: (a) $\mu = 0.3$ (b) 1.53 m**

## 8 Conditional binomial probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, 0.3)$. Find $P(X = 7 \mid X \geq 4)$ to 3 significant figures.

$P(X = 7 \mid X \geq 4) = \frac{P(X = 7)}{P(X \geq 4)}$ (since $X = 7$ implies $X \geq 4$)

$P(X = 7) = 0.164$

$P(X \geq 4) = 1 - P(X \leq 3) = 0.893$

Ratio = 0.184

**Answer: 0.184**

## 9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1$: $\mathbf{r} = \begin{pmatrix} -7 \\ 8 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$l_2$: $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$

Determine whether $l_1$ and $l_2$ intersect. If they do, find the point of intersection.

Equate components: $-7 + 2\lambda = 1 + 2\mu$; $8 - 1\lambda = 5 + 0\mu$; $8 - 2\lambda = 0 - 2\mu$

Solving: $\lambda = 3$, $\mu = -1$, and all three equations are satisfied

Point of intersection = $(-1, 5, 2)$

**Answer: Intersect at $(-1, 5, 2)$**

## 10 Transformations from a graph (All boards)

The diagram shows the curve $y = f(x)$, with turning points A and B.

Find the coordinates of the turning points of

(a) $y = f(x) - 4$ (b) $y = 3f(x)$

(a) $y = f(x) - 4$: A → (2, −6), B → (4, 2)

(b) $y = 3f(x)$: A → (2, −6), B → (4, 18)

**Answer: (a) (2, −6), (4, 2); (b) (2, −6), (4, 18)**