Day 94 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Fixed point iteration (All boards)

The equation $e^x = 5x + 6$ has a root α. Use the iteration $x_{n+1} = \ln(5x + 6)$ with $x_0 = 2$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(5 \times 2 + 6) = 2.7726$

$x_2 = 2.9889$, $x_3 = 3.0419$

Continuing, the values settle at 3.0583, so $\alpha = 3.06$

Answer: $x_1 = 2.7726$, $x_2 = 2.9889$, $x_3 = 3.0419$; $\alpha = 3.06$

2 Line meets curve: length of chord (All boards)

The line $y = -2x + 3$ meets the curve $y = x^2 + x - 1$ at A and B.

(a) Find the coordinates of A and B.

(b) Find the exact length AB.

$x^2 + x - 1 = -2x + 3$ gives $x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$: $x = -4$ or 1

A$(-4, 11)$, B$(1, 1)$

$AB^2 = 5^2 + (-10)^2 = 125$

Answer: A$(-4, 11)$, B$(1, 1)$; AB $= 5\sqrt{5}$

3 Change of sign to locate a root (All boards)

$f(x) = \ln x + 4x - 10$.

Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

$f(x)$ is continuous on the interval

$f(2) = -1.31 < 0$

$f(3) = 3.10 > 0$

A change of sign, so a root lies in $[2, 3]$

Answer: Sign change: $f(2) = -1.31$, $f(3) = 3.10$

4 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 31 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{5}{12}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Horizontal component $= 31 \times \frac{12}{13} = \frac{372}{13}$; vertical component $= 31 \times \frac{5}{13} = \frac{155}{13}$

(a) Vertical: $0 = \frac{155}{13}t - 4.9t^2$, so $t = \frac{1550}{637}$

(b) At the top: $0 = \left(\frac{155}{13}\right)^2 - 2 \times 9.8 \times H$, so $H = \frac{120125}{16562}$

(c) OA $= \frac{372}{13} \times \frac{1550}{637}$

Answer: (a) 2.43 s (b) 7.25 m (c) 69.6 m

5 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 58 items.

Use linear interpolation to estimate the 90th percentile.

Position $= 0.9 \times 58 = 52.2$

Cumulative frequency before the class $100 \leq t < 120$ is 52

Estimate $= 100 + \frac{52.2 - 52}{6} \times 20$

$= 101$

Answer: 101

6 Combined inequalities (All boards)

(a) Solve $3x - 5 \leq -6$.

(b) Solve $x^2 + x - 6 \geq 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $3x - 5 \leq -6$ so $x \leq -\frac{1}{3}$

(b) $(x + 3)(x - 2) \geq 0$: critical values -3 and 2

(b) gives $x \leq -3$ or $x \geq 2$

(c) Overlap of the two sets.

Answer: (a) $x \leq -\frac{1}{3}$ (b) $x \leq -3$ or $x \geq 2$ (c) $x \leq -3$

7 Exponential growth and decay modelling (All boards)

A radioactive sample decays so that its mass is $M = M_0e^{-kt}$ grams, with t in days.

Its half-life is 5 days. Show that $k = \frac{\ln 2}{5}$ and find how long it takes for 95% of the sample to decay.

Half-life: $e^{-5k} = \frac{1}{2}$, so $5k = \ln 2$ and $k = \frac{\ln 2}{5}$

95% decayed means 5% remains: $e^{-kt} = 0.05$

$kt = -\ln 0.05 = \ln 20$, so $t = \frac{5 \ln 20}{\ln 2} = 21.6$ days

Answer: t = 21.6 days

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.3, independently. A box of 9 parts is rejected if it has at least 2 defective. From 8 such boxes, find the probability that exactly 4 are rejected (3 s.f.).

$Y \sim B(9, 0.3)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.196 = 0.804$

$W \sim B(8, 0.804)$ is the number rejected

$P(W = 4) = C(8, 4)(0.804)^4(1 - 0.804)^4 = 0.0432$

Answer: 0.0432

9 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan \alpha = \frac{4}{3}$, and passes through A, 36 m horizontally from O and 25.5 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $36 = U \cos \alpha \times t$, so $t = 36 \div (0.6\ U)$

Vertical: $25.5 = U \sin \alpha \times t - 4.9t^2$

Eliminate t: $y = x \tan \alpha - 4.9x^2 \div (U^2 \cos^2 \alpha)$

$25.5 = 48 - 4.9 \times 36^2 \div (0.36\ U^2)$

$U^2 = 17640 \div 22.5 = 784$, $U = 28$

Answer: 28 m s^{-1}

10 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use $x = 0.005$ to estimate $\sqrt{99}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -2x$

(a) $1 - x - \frac{1}{2}x^2$

$x = \frac{1}{200}$: $99 = 10^2(1 - \frac{1}{100})$ so $\sqrt{99} = 10 \times (1 - \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{200} - \frac{1}{80000}) = 9.949875$

Answer: (a) $1 - x - \frac{1}{2}x^2$ (b) 9.9499